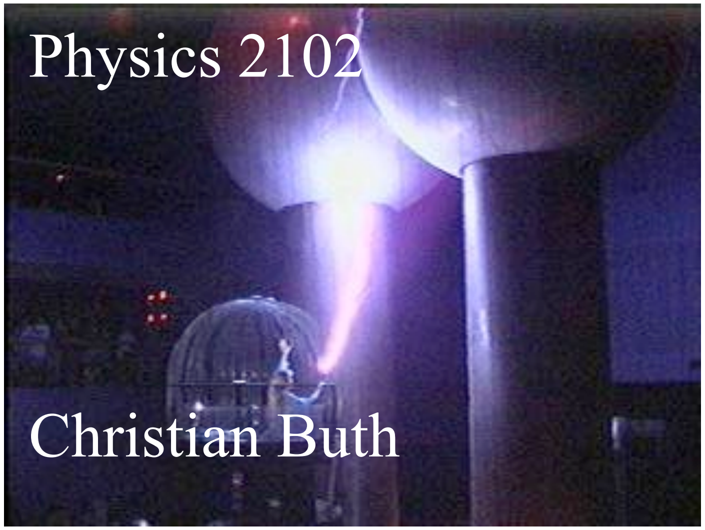
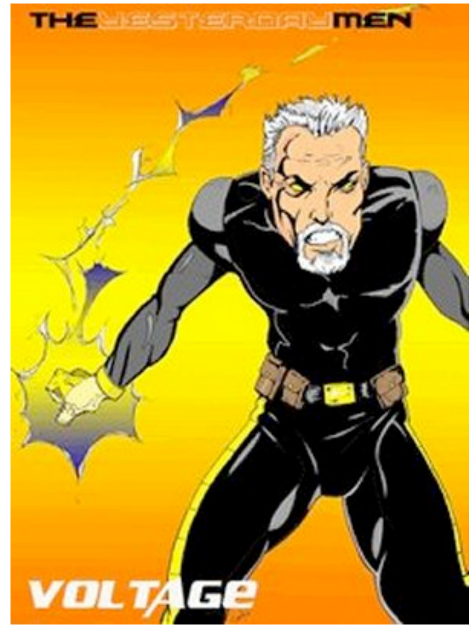
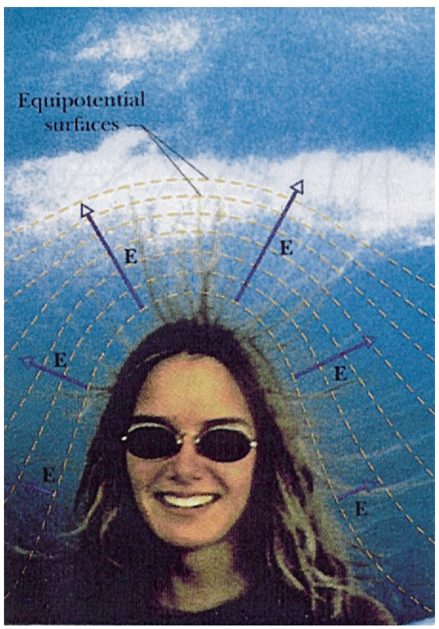


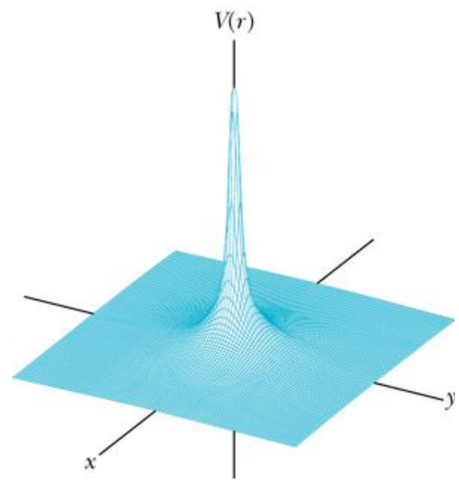




# Physics 2102
# Lecture 9

## Electric Potential 2

Version: 02/02/2009



# Review

- **Electric potential**: work needed to bring +1C from infinity; unit: V = Volt
- Work equals minus applied work equals potential energy difference

$$\Delta U = U_f - U_i = W_{app} = -W = q\Delta V$$

- **Equipotential surface**: constant potential, electric field lines are **perpendicular**
- Electric force is **conservative**: electric potential uniquely defined -- independent of path!

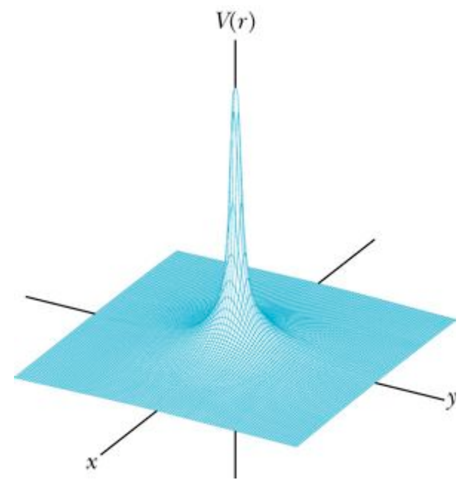
# Electric Potential of a Point Charge

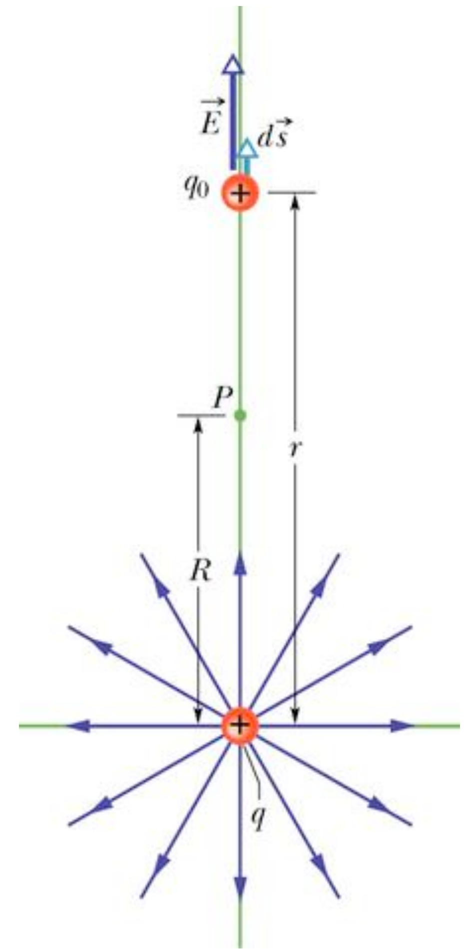
$$V = -\int_i^f \vec{E}\cdot \mathrm{d}\vec{s} = -\int_\infty^P \vec{E}\cdot \mathrm{d}\vec{s} = -\int_\infty^R (-E)\,\mathrm{d}s$$

$$= -\int_\infty^R (-E)(-\mathrm{d}r) = -\int_\infty^R E\,\mathrm{d}r = -\int_\infty^R k\frac{q}{r^2}\,\mathrm{d}r$$

$$= \left[+k\frac{q}{r}\right]_\infty^R = +k\frac{q}{R}$$

- A positive $Q$ produces a positive $V$
- A negative $Q$ produces a negative $V$



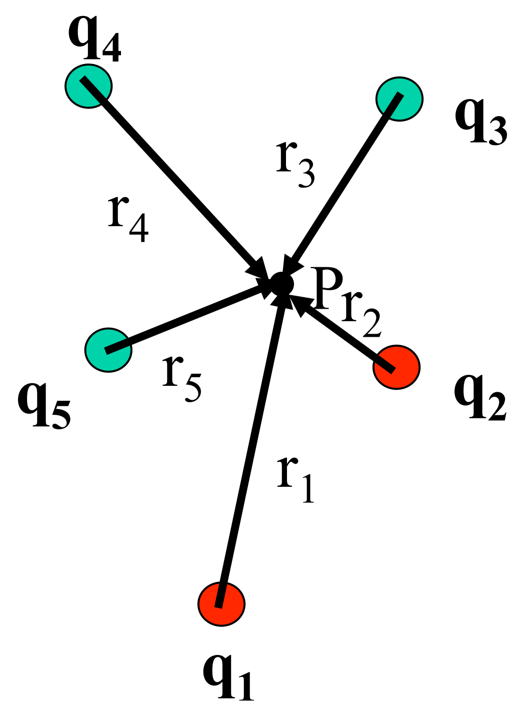



# Electric Potential of Many Point Charges

- Electric potential is a **scalar** not a vector
- Just calculate the potential due to each individual point charge, and add together:

$$V = \sum_i k \frac{q_i}{r_i}$$

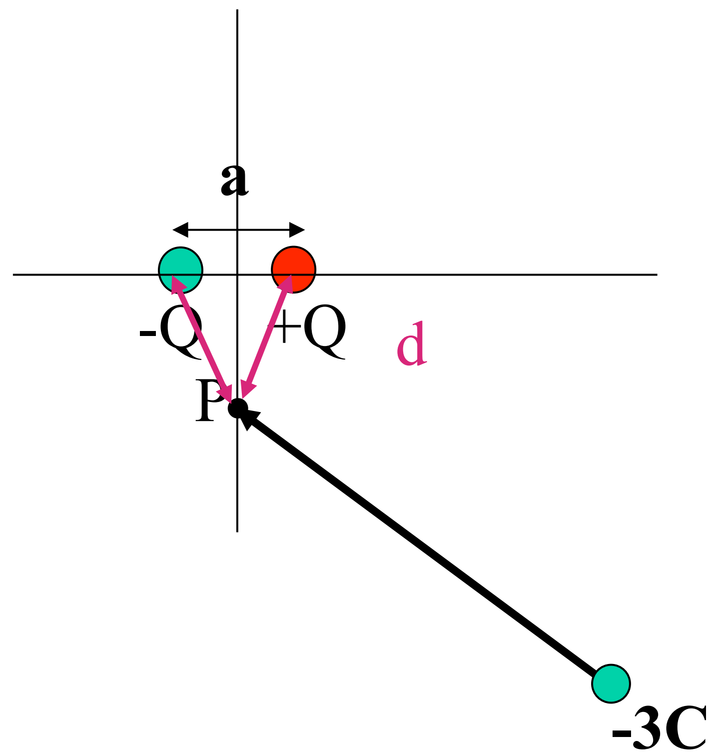
- Follows from the **superposition principle** for the electric field and the linearity of the integration operator



# Electric Potential on Perpendicular Bisector of Dipole

You bring a charge of $Q_o = –3C$ from infinity to a point *P* on the perpendicular bisector of a dipole as shown. Is the work that you do:

a) Positive?
b) Negative?
c) Zero?

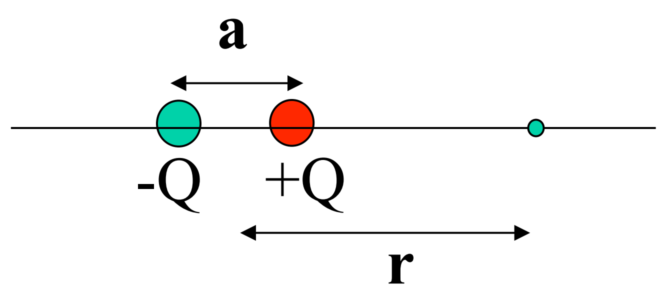



$$U = Q_oV = Q_o(-Q/d + Q/d) = 0$$

# Electric Potential of a Dipole (on axis)

What is $V$ at a point at an **axial distance** $r$ away from the **midpoint** of a dipole (on side of positive charge)?

$$V = k\frac{Q}{(r-\frac{a}{2})} - k\frac{Q}{(r+\frac{a}{2})}$$



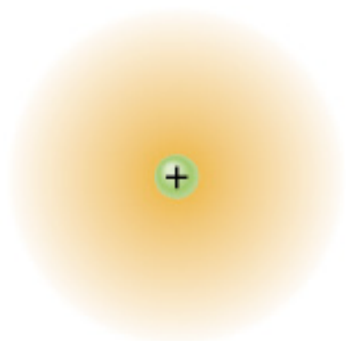

$$= kQ\left(\frac{(r+\frac{a}{2})-(r-\frac{a}{2})}{(r-\frac{a}{2})(r+\frac{a}{2})}\right)$$

$$= \frac{Qa}{4\pi\varepsilon_0(r^2-\frac{a^2}{4})}$$

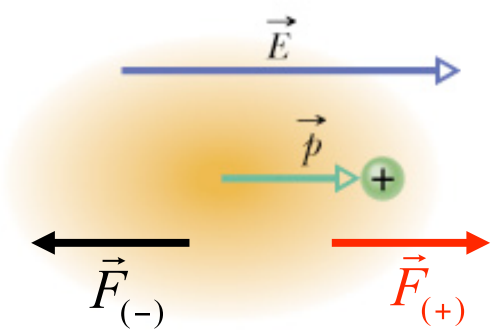
Far away, when r >> a:

$$V = \frac{p}{4\pi\varepsilon_0 r^2}$$

# Induced Dipole Moment



(a)

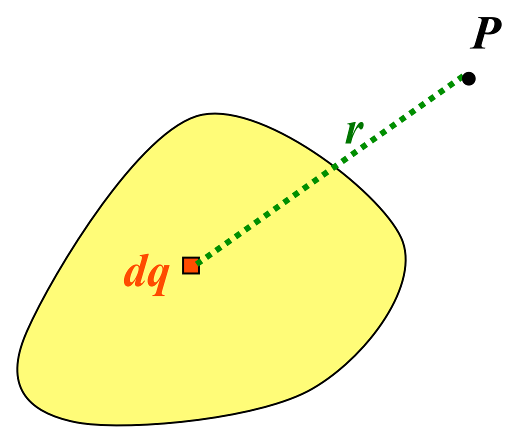



(b)

- **Polar molecules** like $H_2O$ have a permanent dipole moment
- **Nonpolar molecules** like $O_2$, $N_2$ no permanent dipole moment
- Center of positive and negative charge **coincide**
- Electric field pulls positive and negative centers of charge in **opposite** directions
- This **induced** dipole moment disappears when the field disappears

# Continuous Charge Distributions

- Divide the charge distribution into **differential elements**
- Associate d$q$ with a volume / area / line element via a charge density
- Write down an expression for potential from a typical element — treat as point charge
- Integrate!




$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

# Electric Field & Potential: A Simple Relationship!

Notice the following:

- Point charge:
  - $E = kQ/r^2$
  - $V = kQ/r$
- Dipole (far away):
  - $E \sim kp/r^3$
  - $V \sim kp/r^2$
- $E$ is given by a **derivative** of $V$!

$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{s}$$

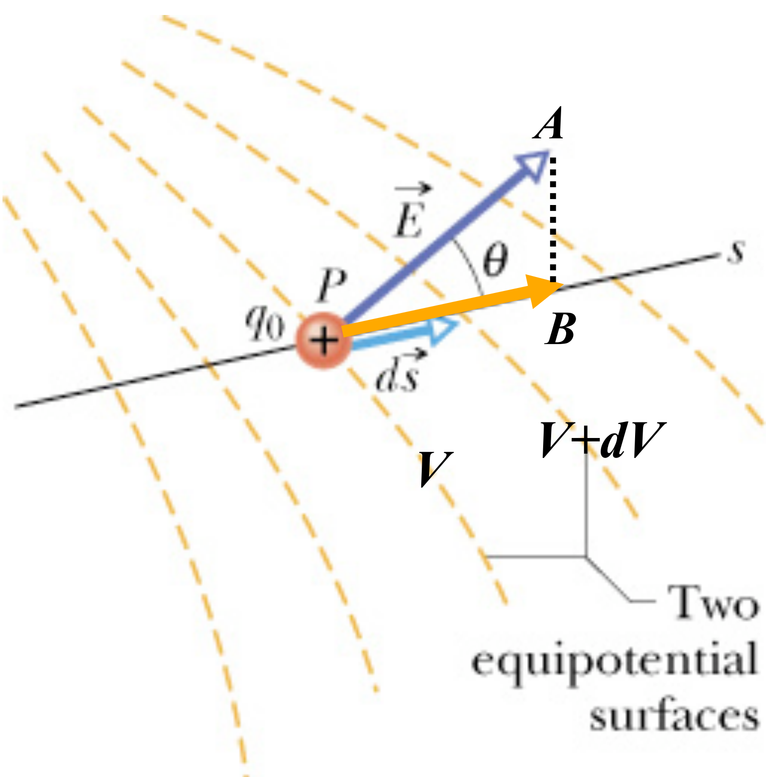
Focus only on a simple case: electric field that points along +x axis but whose magnitude varies with x.

$$E_x = -\frac{dV}{dx}$$

- **Minus** sign!
- Units for $E$ -- VOLTS/METER (V/m)

# Electric Field from Potential 1

- **Reverse problem**: given potential V
- Move $q_0 > 0$ from $V$ to $V + \mathrm{d}V$ along $\mathrm{d}\vec{s}$




The work done by the electric field is given by:

$W = -q_0 dV$ (eq. 1).

Also $W = F ds \cos\theta = E q_0 ds \cos\theta$ (eq. 2)

If we compare these two equations we have:

$$E q_0 ds \cos\theta = -q_0 dV \rightarrow E\cos\theta = -\frac{dV}{ds}.$$

From triangle $PAB$ we see that $E\cos\theta$ is the component $E_s$ of $\vec{E}$ along the direction $s$.

Thus: $E_s = -\frac{\partial V}{\partial s}.$

# Electric Field from Potential 2

The component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes with distance in this direction.

If we take $s$ to be the $x$-, $y$-, and $z$-axes we get:

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

$$E_s = -\frac{\partial V}{\partial s}$$

If we know the function $V(x, y, z)$

we can determine the components of $\vec{E}$

and thus the vector $\vec{E}$ itself :

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

# Summary

- Potential by a **point charge**: $V=kq/r$
- Potential of a continuous **charge distribution**: $V=\int kdq/r$
- Potential of a **dipole** (with angular dependence):

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

- Electric field follows from potential by **derivation**:

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$