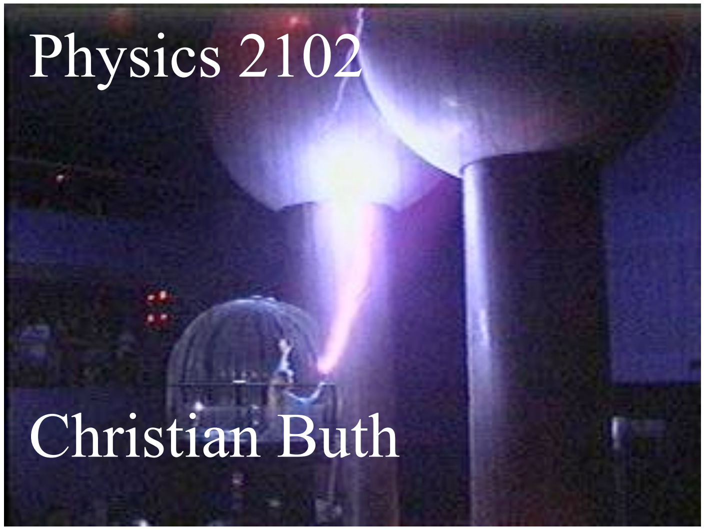


Flux Capacitor (Operational)

Physics 2102
Lecture 7

Gauss' Law 2

Carl Friedrich Gauss
1777-1855

Version: 01/28/2009

Review

- **Gauss' Law**: the flux through a closed (Gaussian) surface is the total charge divided by the permittivity constant:

$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

- With **symmetry**, Gauss' law provides direct way to the **electric field**

- Field **inside** conductors is zero; excess charges are always on the **surface**; field on the surface is **perpendicular** and $E=\sigma/\varepsilon_0$

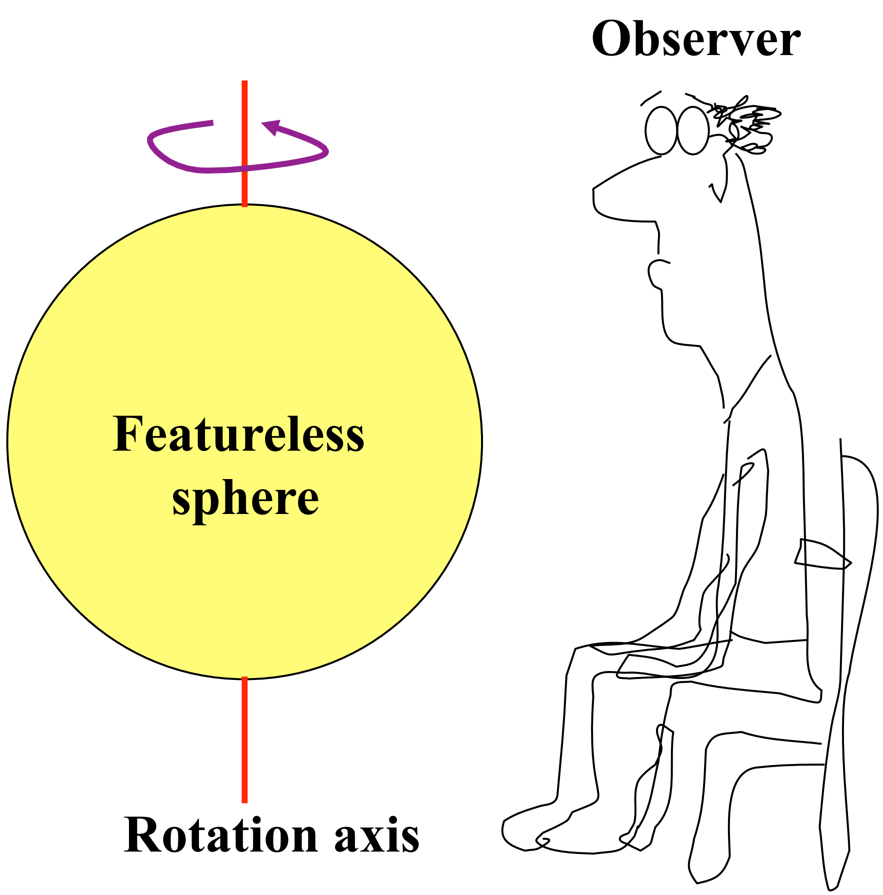
Symmetry

- Particular mathematical **symmetry operation** (e.g., rotation, translation, …)

- An object is symmetric to an observer, if the object looks the **same** before and after the operation

- Symmetry is a **primitive notion** and as such is very powerful

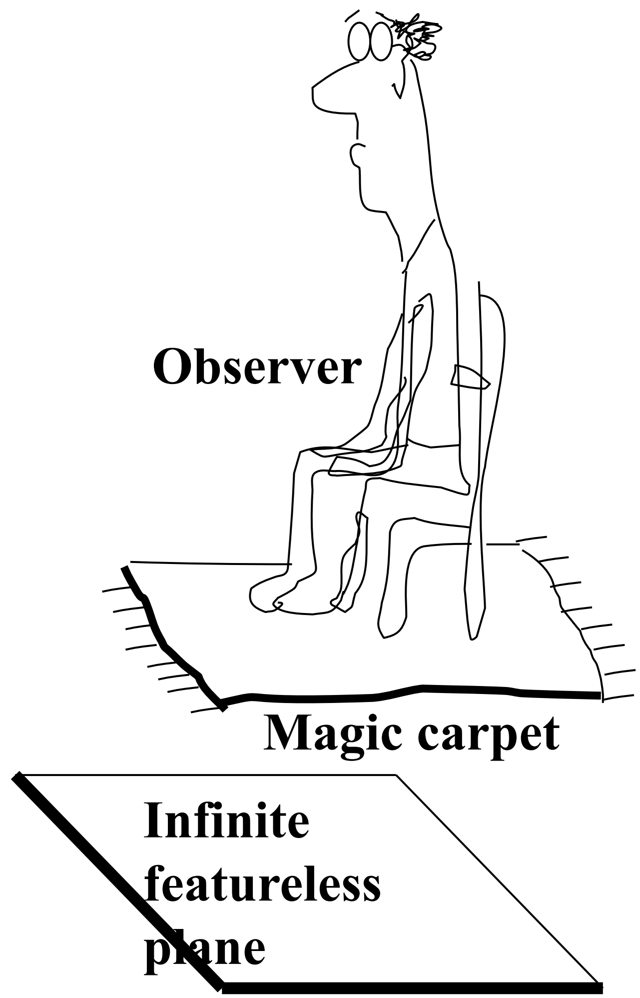
Spherical Symmetry



- **Featureless** beach ball
- **Rotate** about a vertical axis that passes through its center
- Observer **cannot** tell whether the sphere has been rotated or not
- Sphere has **rotational symmetry** about the rotation axis

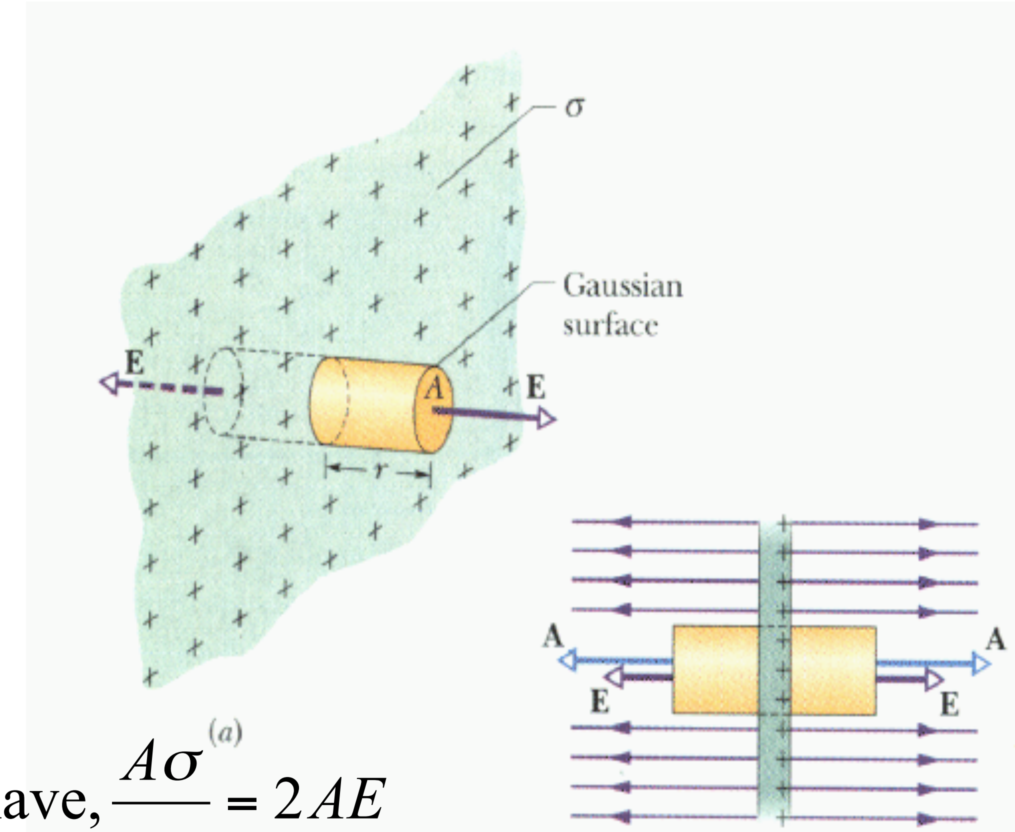
Translational Symmetry

- Infinite **featureless** plane
- Observer takes a **trip** on a magic carpet above the plane
- Observer **cannot** tell whether he has moved or not
- Plane has **translational symmetry**



Gauss' Law: Example

- Infinite INSULATING plane with uniform charge density σ
- E is NORMAL to plane
- Construct Gaussian box as shown

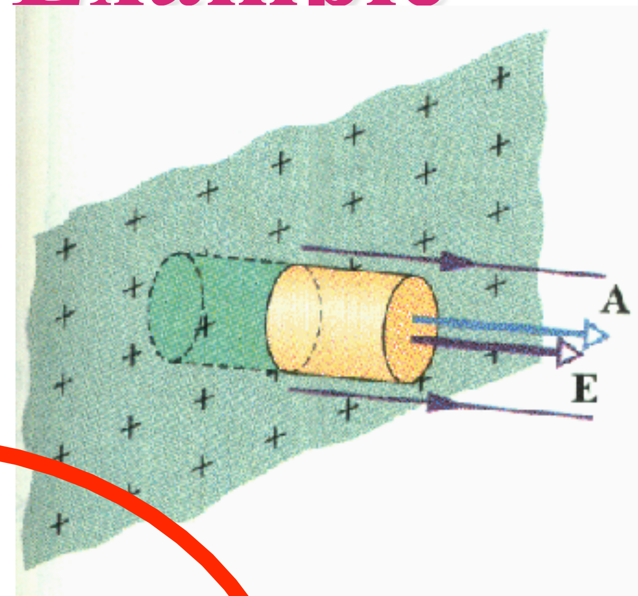


Applying Gauss' law $\frac{q}{\varepsilon_0} = \Phi$, we have, $\frac{A\sigma}{\varepsilon_0} = 2AE$

Solving for the electric field, we get $E = \frac{\sigma}{2\varepsilon_0}$

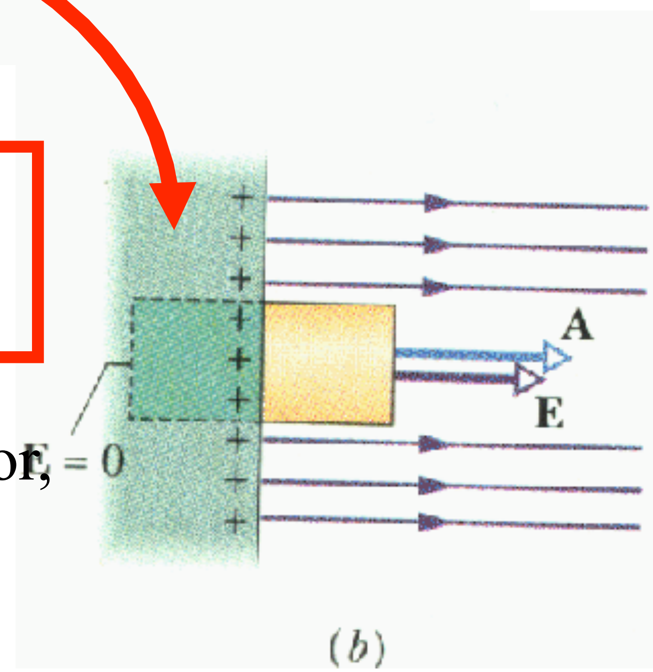
Gauss' Law: Example

- Infinite CONDUCTING plane with uniform areal charge density σ
- E is NORMAL to plane
- Construct Gaussian box as shown.
- Note that E = 0 inside conductor



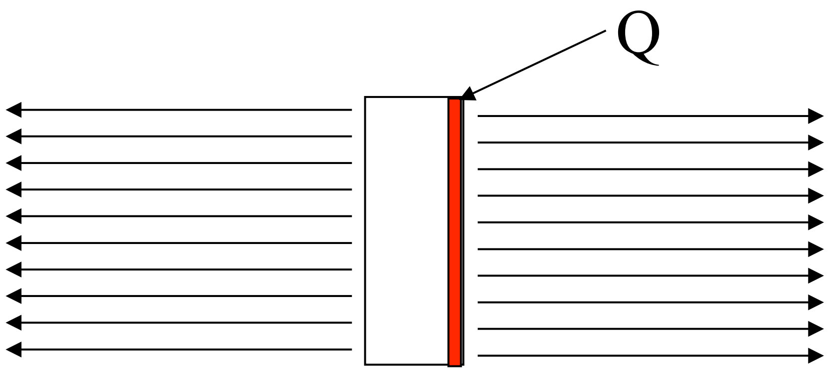
Applying Gauss' law, we have, $\dfrac{A\sigma}{\varepsilon_0} = AE$

Solving for the electric field, we get $E = \dfrac{\sigma}{\varepsilon_0}$



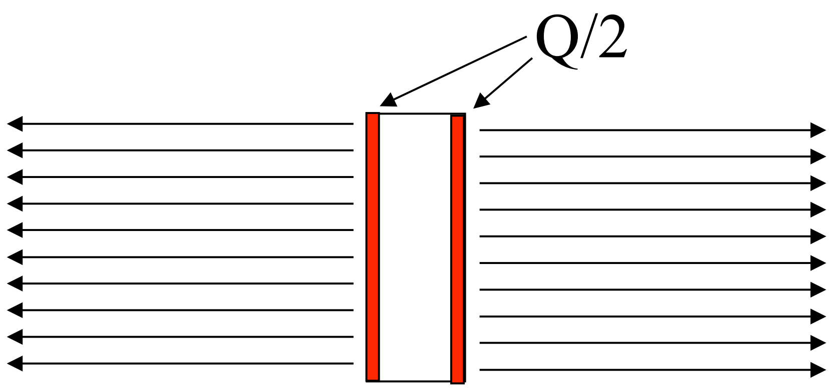
For an insulator, $E=\sigma/2\varepsilon_0$, and for a conductor, $E=\sigma/\varepsilon_0$. Does the charge in an insulator produce a weaker field than in a conductor?

Insulating and conducting planes



$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

Insulating plate: **let** the charge be distributed on **one** surface only

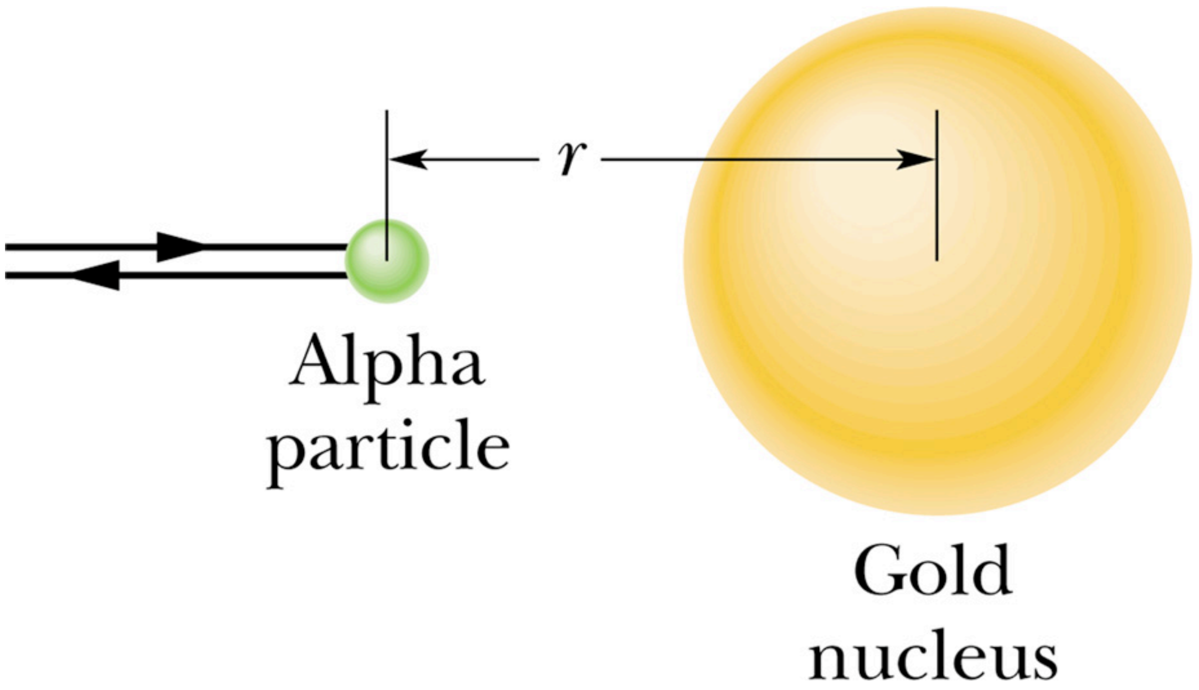


$$E = \frac{\sigma_1}{\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

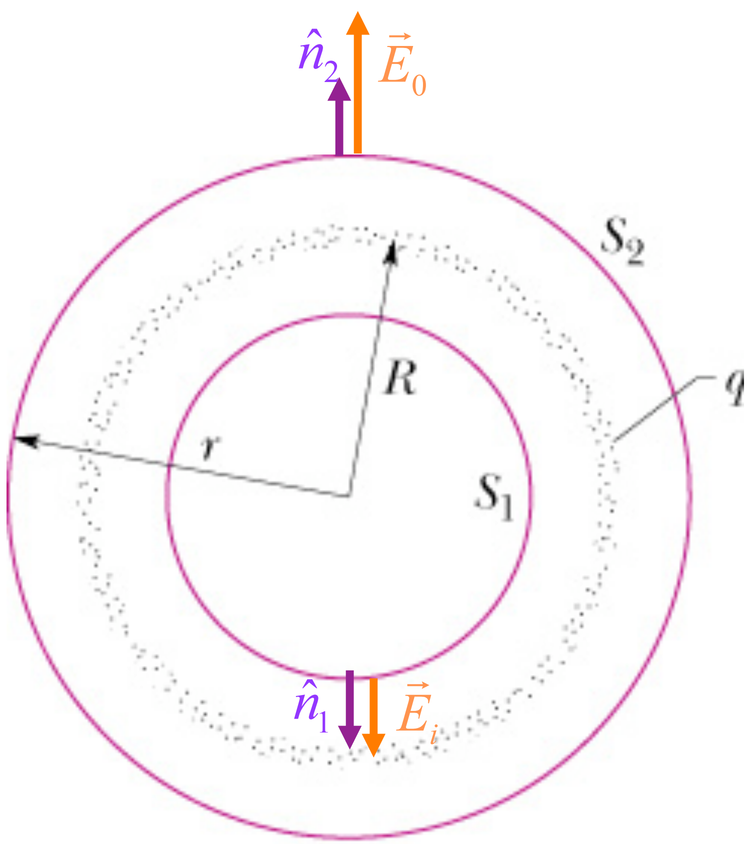
Conducting plate: charge distributed on the outer surfaces

First shell theorem

- A shell of uniform charge effects an outside charge as if the shell was a point charge



Proof of First Shell Theorem



- **Concentric** spherical Gaussian surface outside shell
- The electric field **flux** is

$$\Phi = 4\pi r^2 E_o = \frac{q}{\varepsilon_0}$$

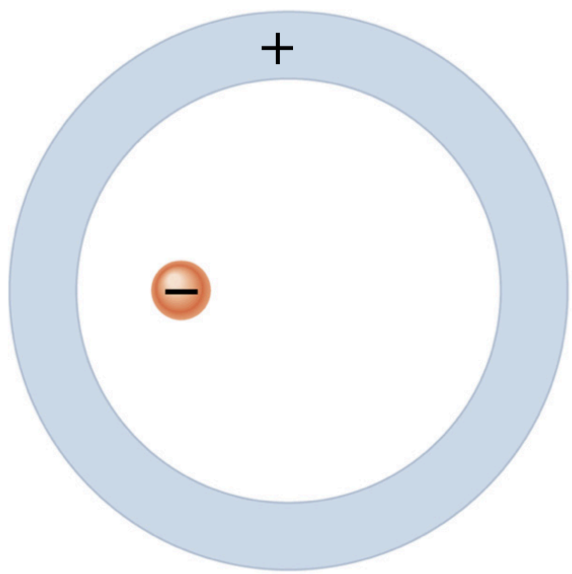
- Thus

$$E_o = \frac{q}{4\pi\varepsilon_0 r^2}$$

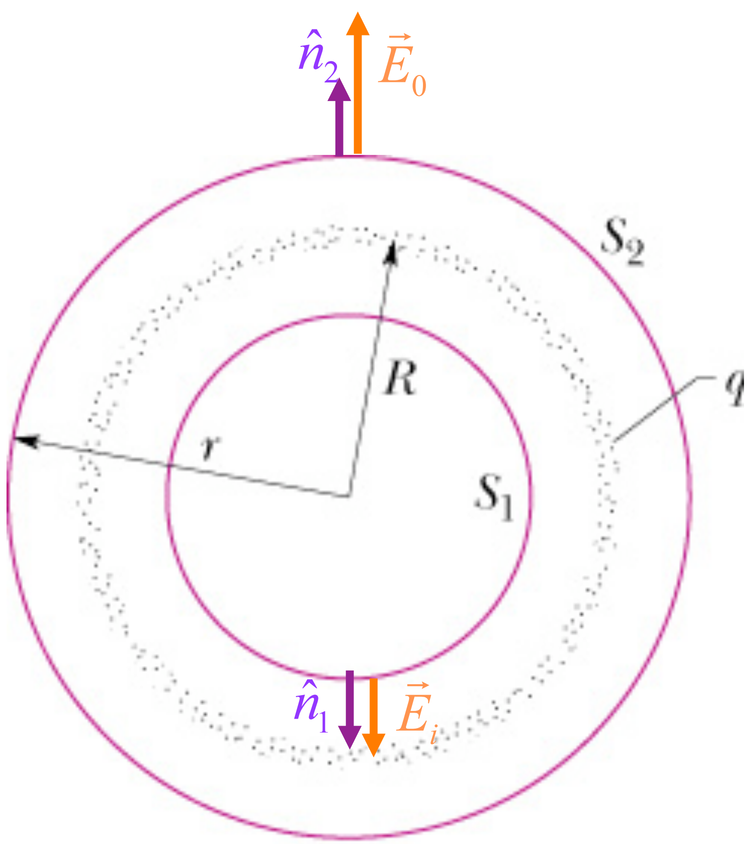
- This is equal to the field of a **point charge** in the center of the shell

Second Shell Theorem

- If a charged particle is in a shell of uniform charge then there is no electrostatic effect due to the shell on the particle



Proof of Second Shell Theorem



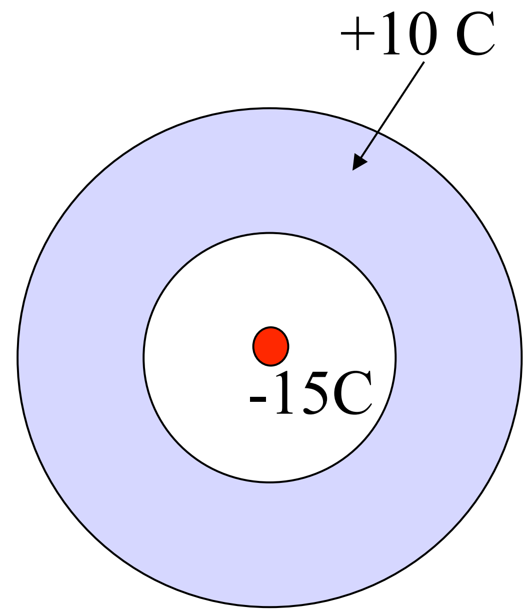
- **Concentric** spherical Gaussian surface inside shell
- The electric field **flux** is

$$\Phi = 4\pi r^2 E_{\mathrm{i}} = 0$$

- Thus

$$E_{\mathrm{i}} = 0$$

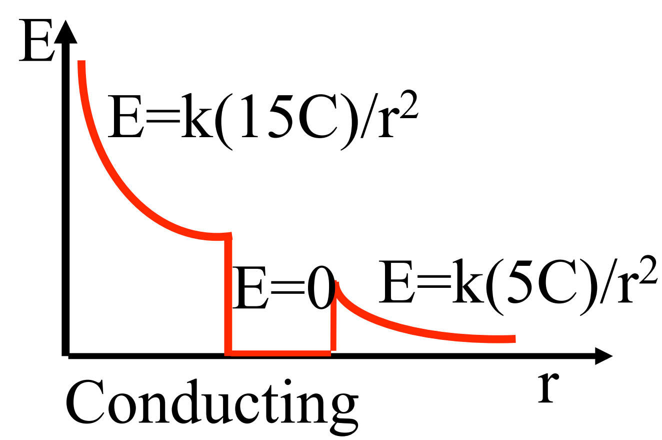
Example: Shell Theorem



A spherical shell has a charge of +10C and a point charge of –15C at the center. What is the electric field produced OUTSIDE the shell?

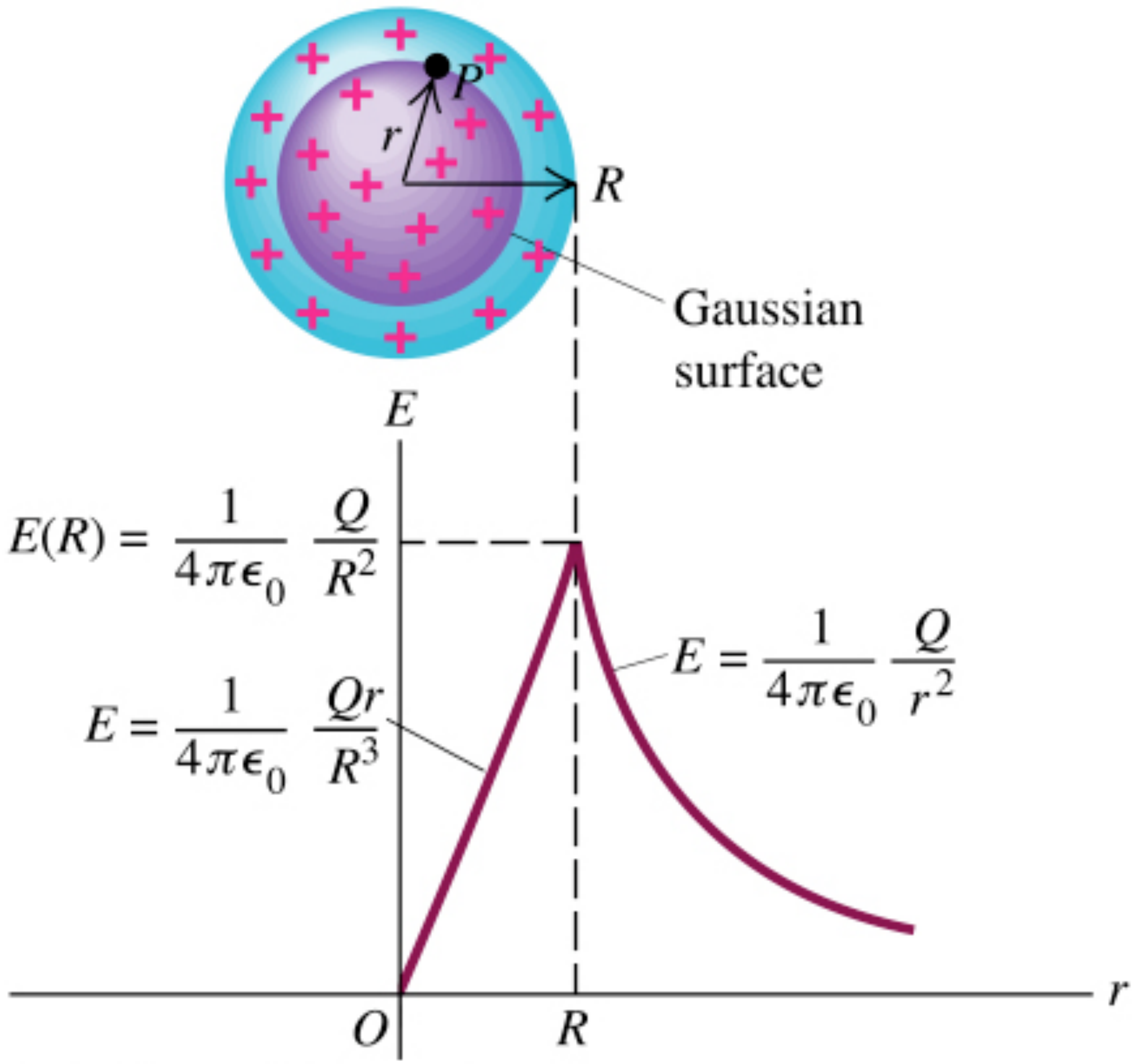
If the shell is conducting:

And if the shell is insulating?



Charged Shells
Behave Like a Point Charge of
Total Charge "Q" at the Center
Once Outside the Last Shell!

Electric Fields With Spherical Symmetry: Insulating



Summary

- Gauss' law provides a very direct way to compute the electric field in situations with symmetry
- Field of an insulating plate: $\sigma/2\varepsilon_{0,}$; of a conducting plate: $\sigma/\varepsilon_{0.}$.
- Properties of conductors: field inside is zero; excess charges are always on the surface; field on the surface is perpendicular and $E=\sigma/\varepsilon_{0.}$