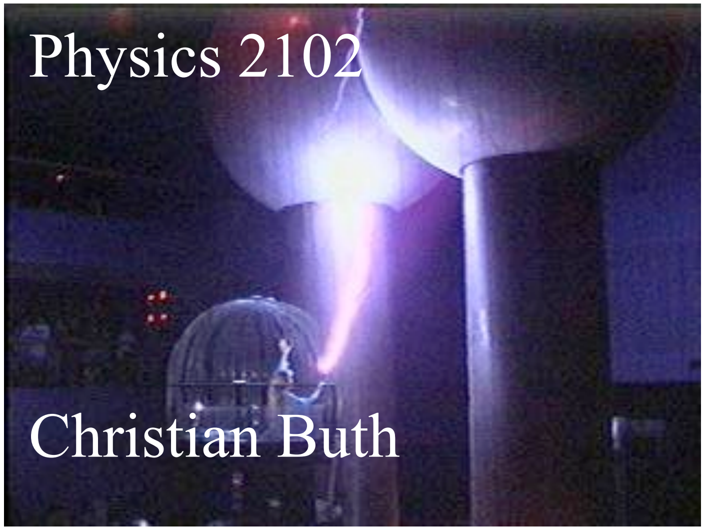


Flux Capacitor (Operational)

Physics 2102
Lecture 6

Gauss' Law 1

Carl Friedrich Gauss
1777-1855

Version: 01/26/2009

Review

- An **electric dipole** in an electric field rotates to align itself with the field

- **Torque:** $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$

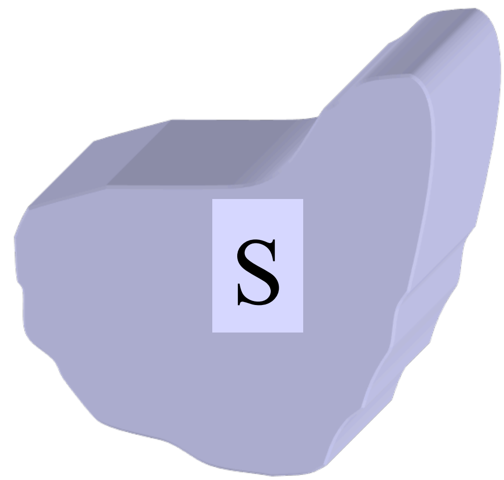
- Work done by electric field is negative of work by an **external agent**: $W_a = \Delta U = -W$

- **Electric flux** is $\Phi = \int \mathbf{E} \cdot d\mathbf{A}$

- The **area vector** always points outward

Gauss' Law

- Consider any **arbitrary closed** surface S, "Gaussian surface" -- note: this does **not** have to be a "real" physical object
- The total electric flux through S is proportional to the **total charge enclosed**
- The results of a complicated integral is a very simple formula: it avoids long calculations



$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

(One of Maxwell's four equations!)

Recipe for Applying Gauss' Law

1. Make a sketch of the charge distribution

2. Identify the symmetry of the distribution and its effect on the electric field

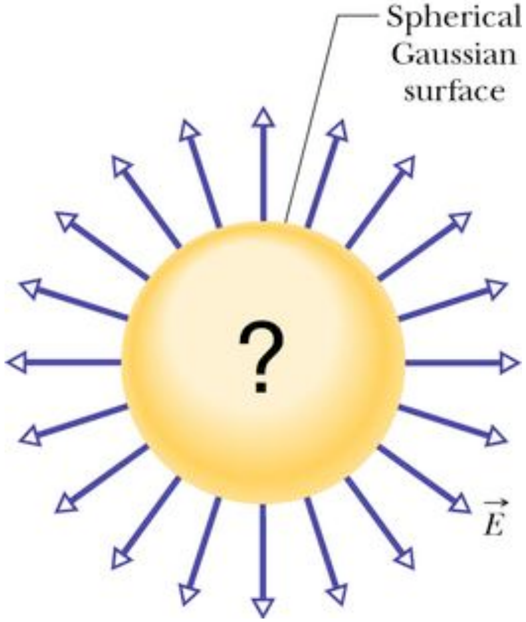
3. Gauss' law is true for **any** closed surface *S*. Choose one that makes the calculation of the flux Φ as easy as possible

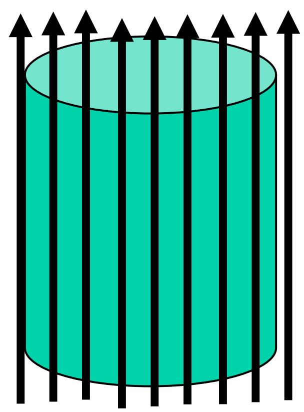
4. Use Gauss' law to determine the, e.g., the enclosed charge vector:

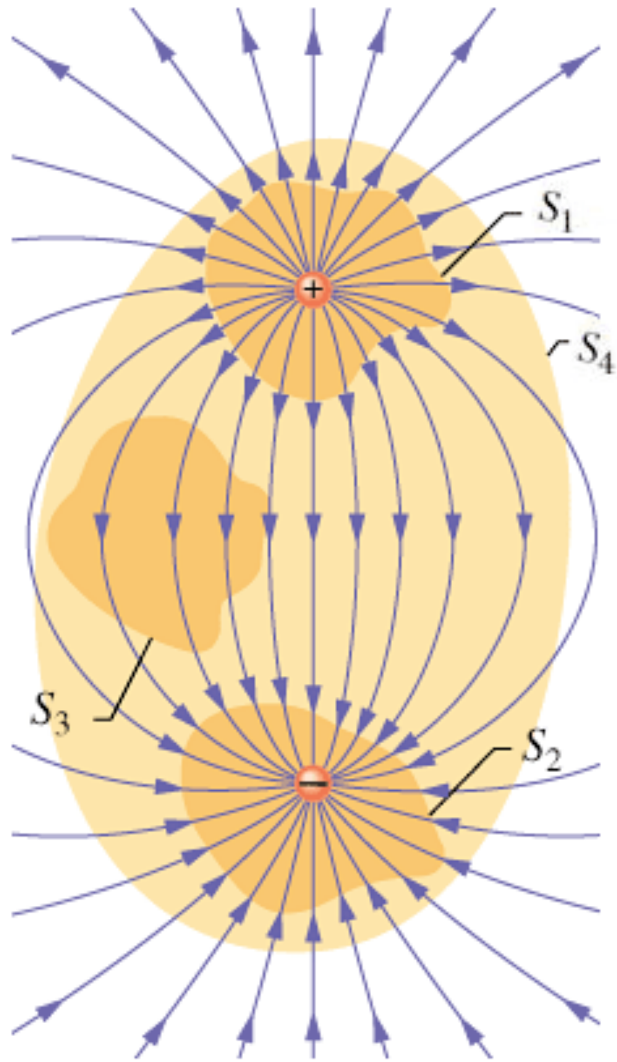
$$\Phi = \frac{q_{\text{enc}}}{\varepsilon_0}$$

Examples

$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

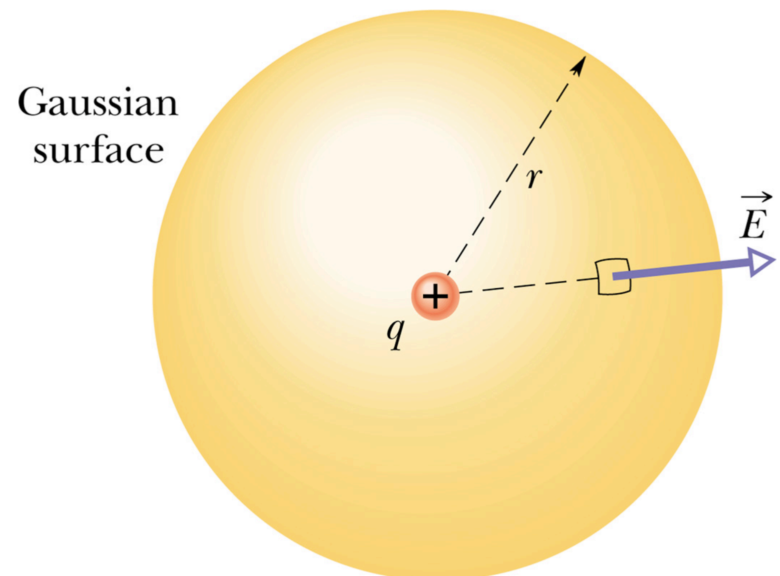






Gauss' Law and Coulomb's Law

- A point charge of q is located at center of the sphere
- Use Gauss' Law to calculate the electric field

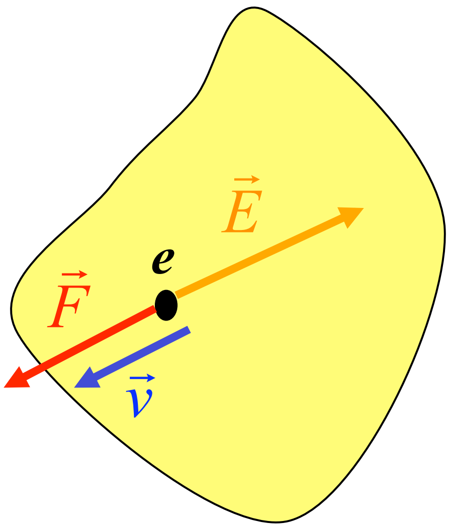


$$\varepsilon_0 \oint_{\text{Surface}} \vec{E} \cdot \mathrm{d}\vec{A} = \varepsilon_0 E \oint_{\text{Surface}} \mathrm{d}A = q$$

$$\varepsilon_0 E(4\pi r^2) = q$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

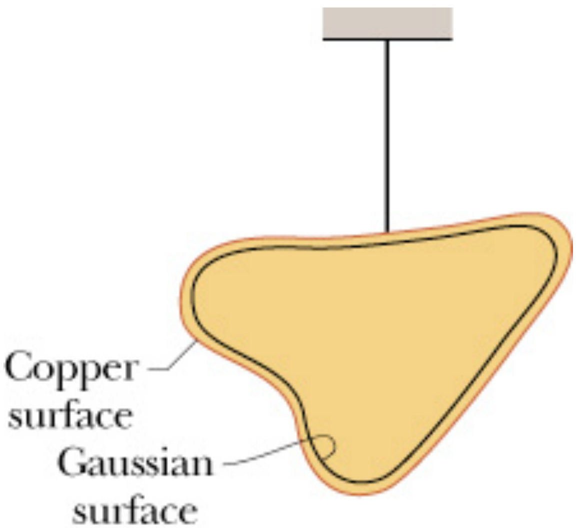
Electric Field Inside a Conductor



- **Conductor** in electrostatic equilibrium; it contains movable electrons
- If there was electric field inside, a **force** would be exerted on electrons
- Electrons would move, i.e., a **perpetual current** would flow
- Current would heat conductor; could be detected by its **magnetic field**
- **No** such current has been observed

The electrostatic electric field $\vec{E}$ inside a conductor is equal to zero.

A Charged Isolated Conductor

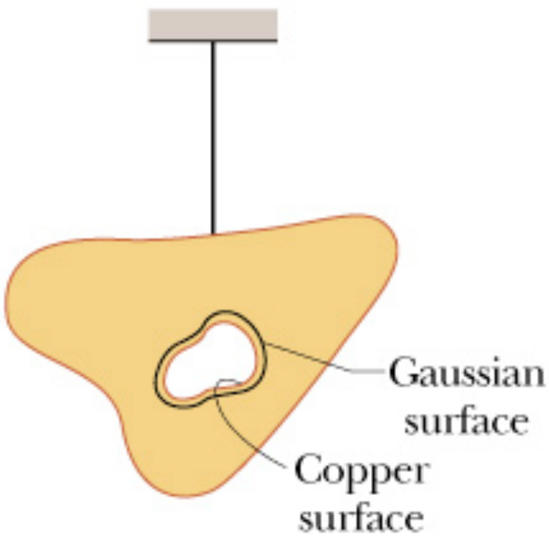


- **Conductor** in electrostatic equilibrium with excess charge *q*
- **Gauss' law**: no field in conductor thus **no net charge** enclosed by Gaussian surface
- Excess charge can only be on the **surface**

No electrostatic charges can exist inside a conductor.

All charges reside on the conductor surface.

Charged Conductor with a Cavity

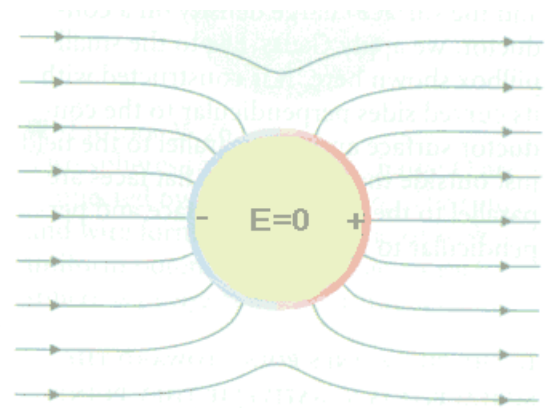


- **Conductor** in electrostatic equilibrium with excess charge q
- Is some excess charge on the **cavity walls**?
- **Gauss' law**: no field in conductor thus **no net charge** enclosed by Gaussian surface
- Excess charge can only be on the **outer surface**

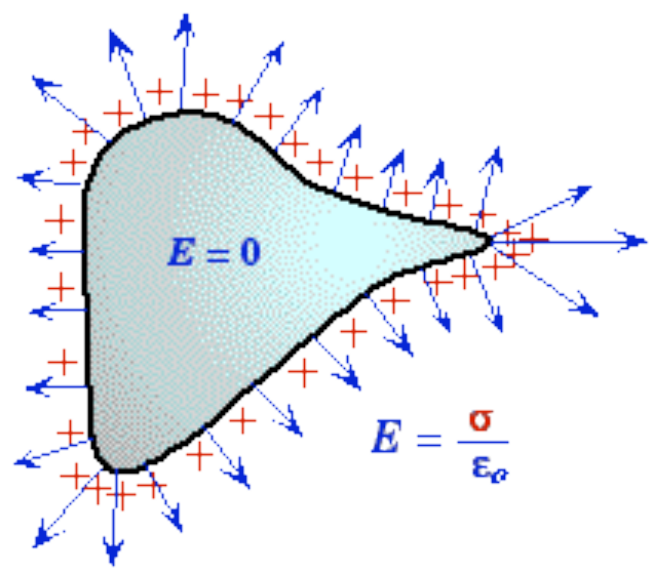
There is no charge on the cavity walls. All the excess charge q remains on the outer surface of the conductor.

Electric Field around conductors

• We assume **electrostatic equilibrium** with excess charge *q*
• Excess charges produce an electric field outside the conductor



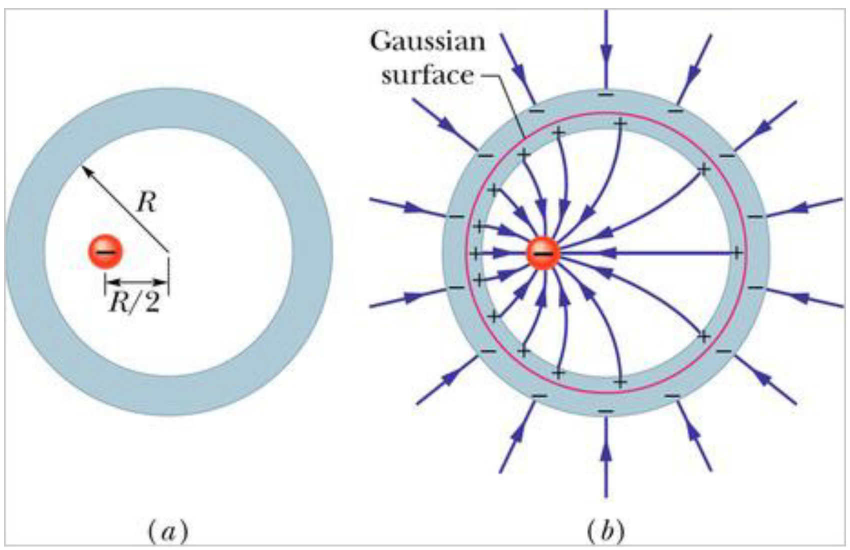
On the surface of conductors in electrostatic equilibrium, the electric field is always perpendicular to the surface

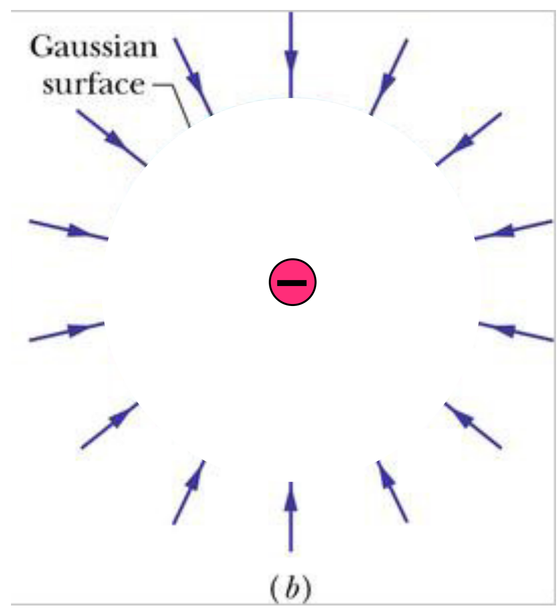


Why? If not, charges on the surface would move due to the tangential component of the electric field

Charges in conductors

- Consider a **conducting shell**, and a negative charge inside the shell
- Charges will be **induced** in the conductor to make the field inside the conductor zero
- Outside the shell, the field is the same as the field produced by a **charge at the center**





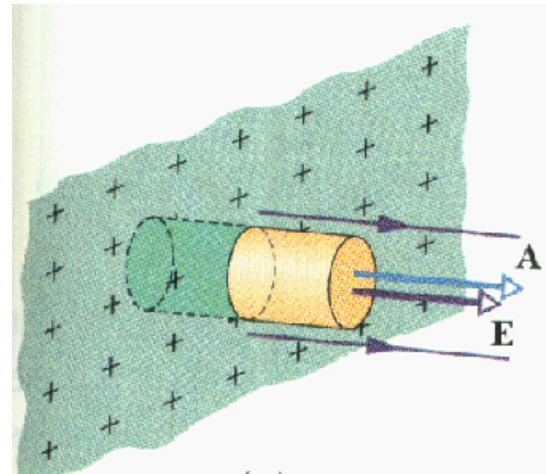
Gauss' Law: Example

- Infinite **conducting** plane with uniform area charge density σ
- Note that E = 0 **inside** conductor
- Applying Gauss' law, we have

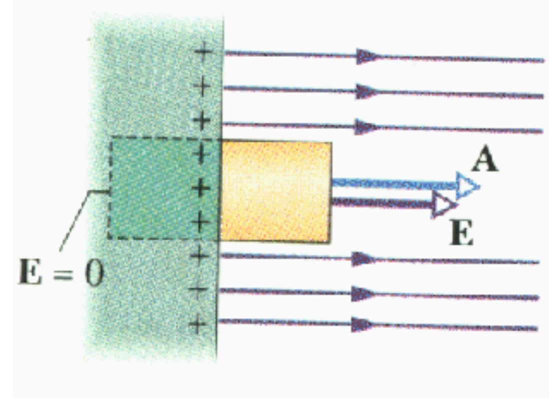
$$\frac{A\sigma}{\varepsilon_0} = AE$$

- Solving for the electric field, we get

$$E = \frac{\sigma}{\varepsilon_0}$$



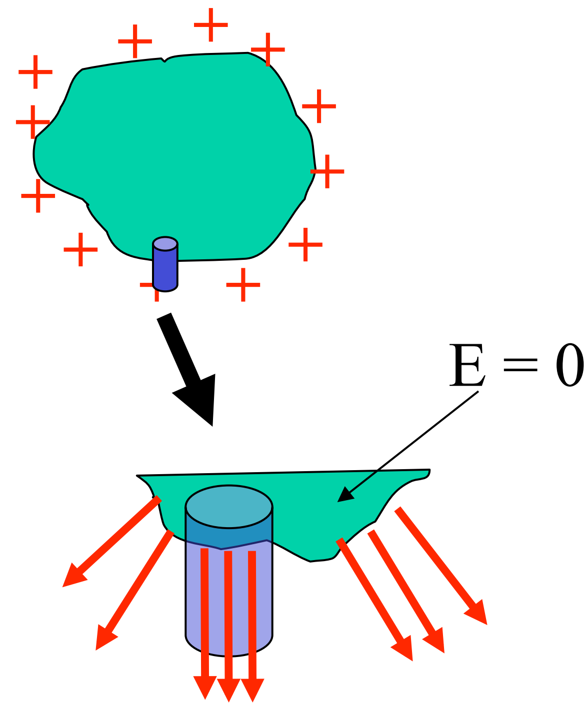
(a)



(b)

Gauss' Law: Conducting Example

- Charged conductor of arbitrary shape: no symmetry; non-uniform charge density
- What is the electric field near the surface where the local charge density is σ?



(a) σ/ε_0 ✓

(b) Zero

(c) $\sigma/2\varepsilon_0$

Applying Gauss' law, we have, $\dfrac{A\sigma}{\varepsilon_0} = AE$

Solving for the electric field, we get $E = \dfrac{\sigma}{\varepsilon_0}$

THIS IS A GENERAL RESULT FOR CONDUCTORS!

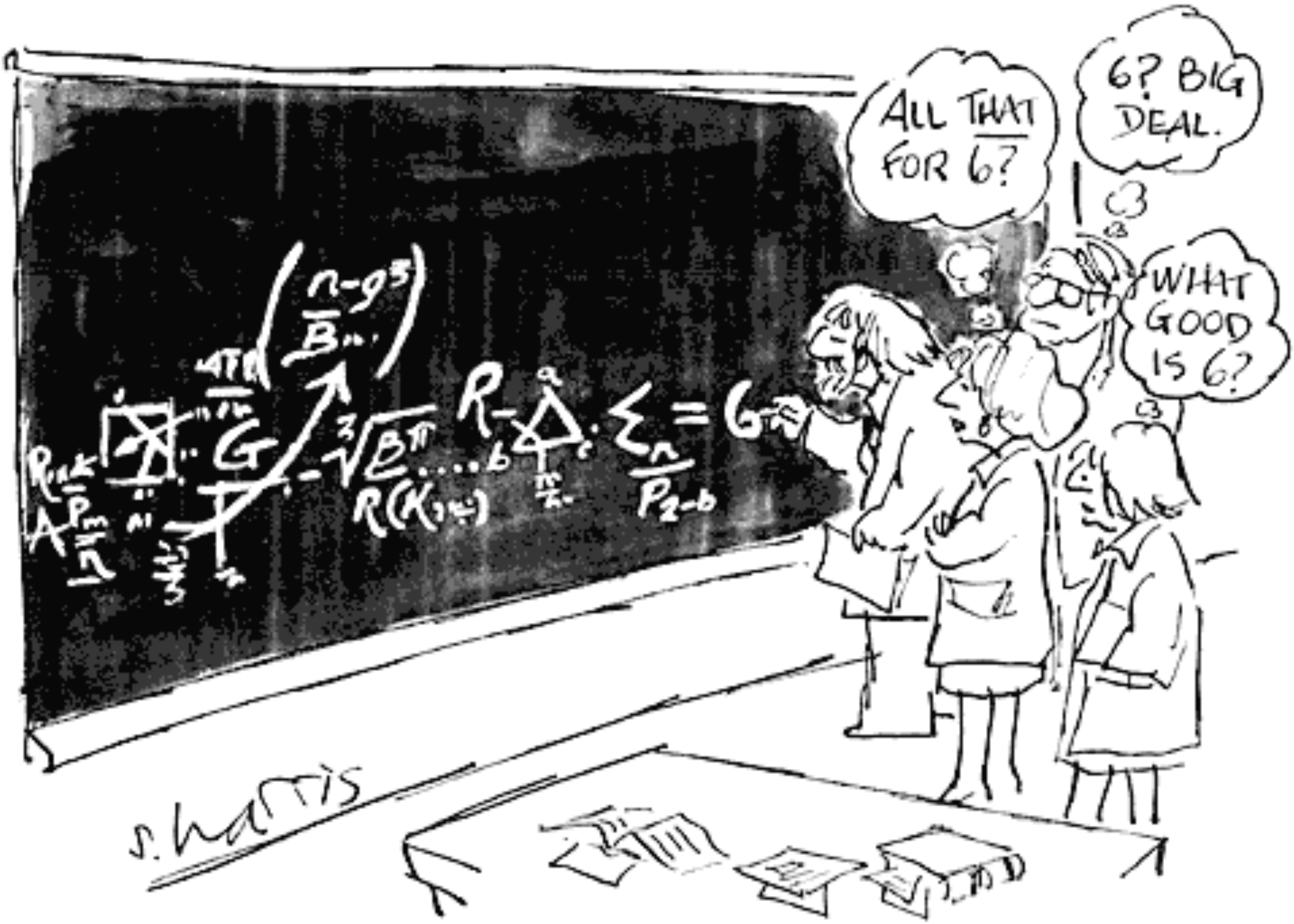
Summary

- **Gauss' Law**: the flux through a closed (Gaussian) surface is the total charge divided by the permittivity constant:

$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

- With **symmetry**, Gauss' law provides direct way to the **electric field**

- Field **inside** conductors is zero; excess charges are always on the **surface**; field on the surface is **perpendicular** and $E=\sigma/\varepsilon_0$


ALL THAT FOR 6?
6? BIG DEAL.
WHAT GOOD IS 6?
S. HARRIS