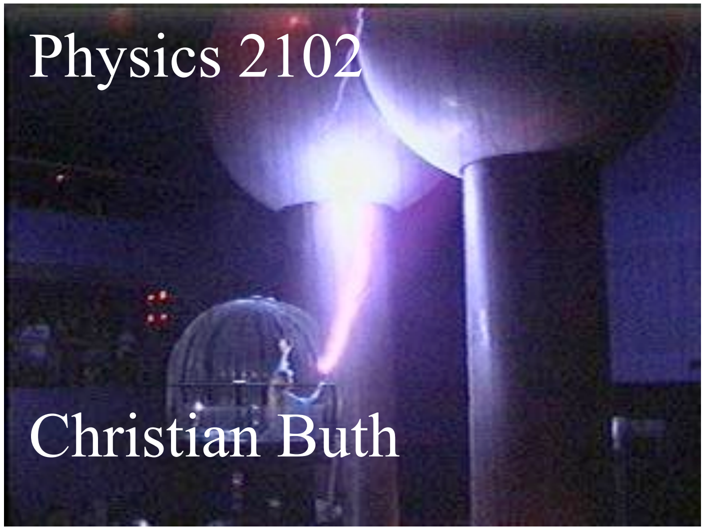
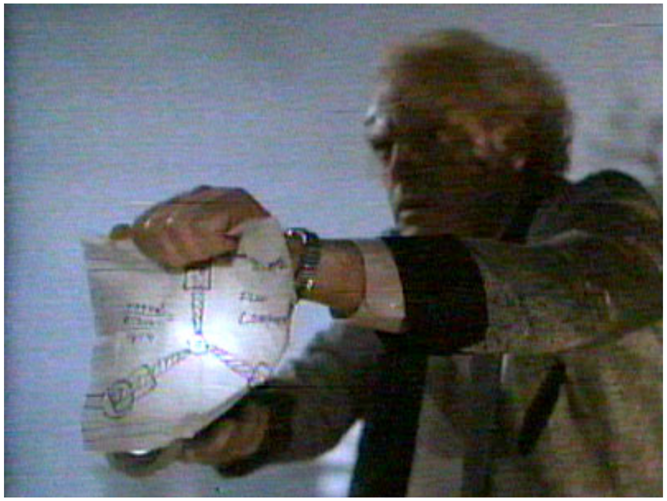
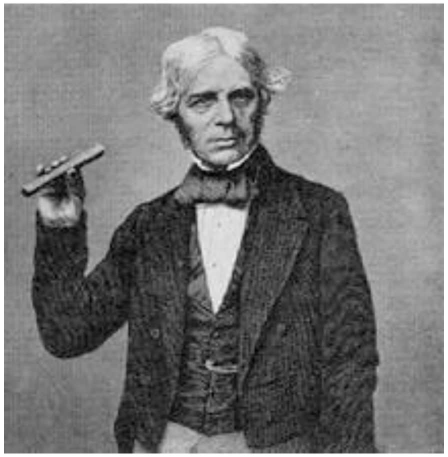




Flux Capacitor (Schematic)

Physics 2102
Lecture 5

Electric Flux



Michael Faraday
1791-1867

Version: 01/23/2009

Review

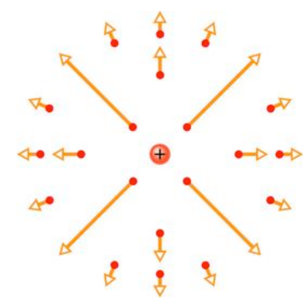
- **Continuous charge distributions** instead of discrete point charges
- Use **calculus** to find electric field from a continuous charge distribution
- For a **known** electric field, determine **force** a on charged particle via $\vec{F} = q\,\vec{E}$
- Electric dipole **aligns along lines** of a uniform electric field
- **Milliken experiment** proves that charge is quantized; yields the magnitude of the **elementary charge**

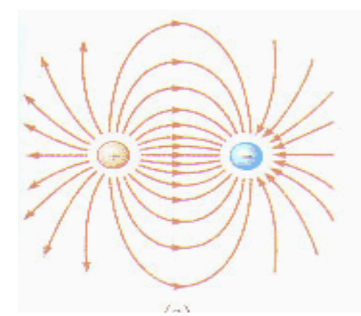
Electric charges and fields

We work with two different kinds of problems, easily confused:

• **Given certain electric charges**, we calculate the electric field produced by those charges (using **E**=kq**r**/r^3 for each charge)

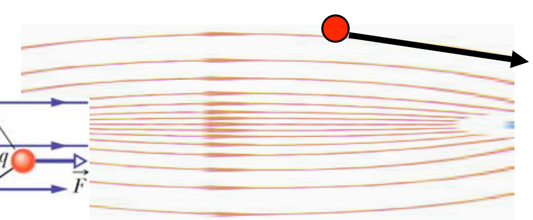
Example: the electric field produced by a single charge, or by a dipole:

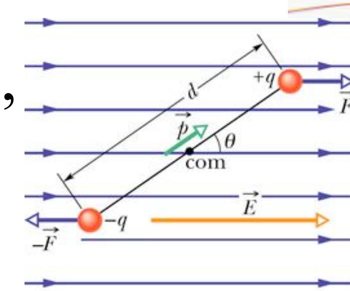




• **Given an electric field**, we calculate the forces applied by this electric field on charges that come into the field, using **F**=q**E**

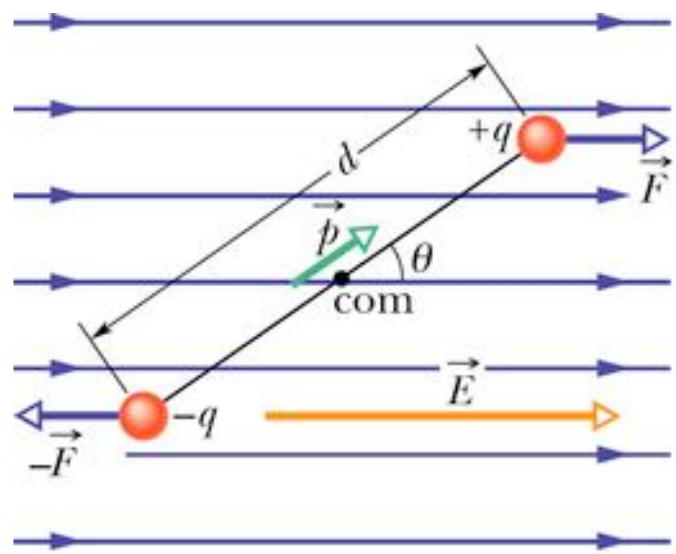
Examples: forces on a single charge when immersed in the field of a dipole, torque on a dipole when immersed in an uniform electric field



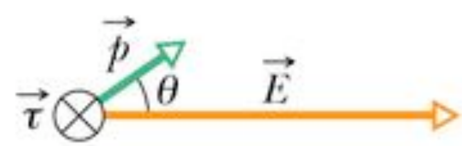


Electric Dipole in a Uniform Field

- **No net force** on dipole; center of mass stays where it is
- Net **torque** $\boldsymbol{\tau}$: into page
- Dipole **rotates** to line up in direction of ***E***
- $|\boldsymbol{\tau}| = 2\,(QE)\,(d/2)\,(\sin\theta)$
  $= (Qd)\,(E)\sin\theta$
  $= |\mathbf{p}|\,E\sin\theta$
  $= |\mathbf{p} \times \mathbf{E}|$
- The dipole tends to "align" itself with the field lines
- What happens if the field is **not uniform**?



(a)



(b)

Potential Energy

- The work W done by the **electric field** is

$$\Delta U = U_f - U_i = \int_i^f F \cdot d\vec{s} = -W$$

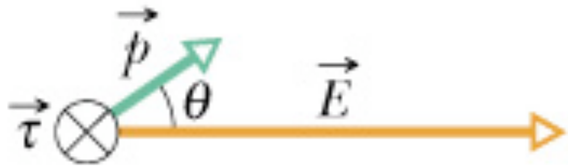
  equals the negative potential energy difference ΔU

- The work done by an **external agent** equals the potential energy difference

$$W_a = \Delta U = -W$$

- The **minus sign** comes from Newton's third law (action equals reaction): the force due to the electric field is of the **same** magnitude but of **opposite** direction than the force exerted by the external agent

Potential Energy of an Electric Dipole in a Uniform Electric Field

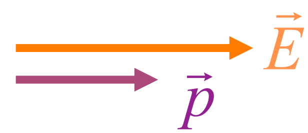


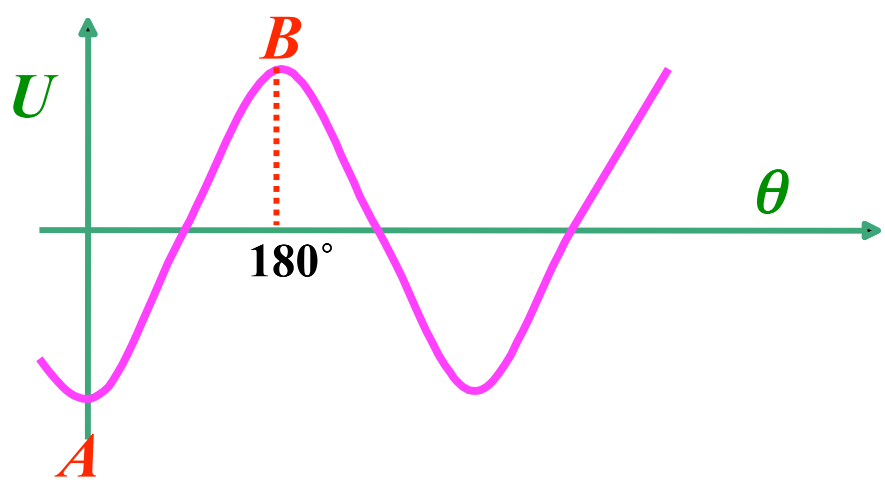
$$U = -pE\cos\theta$$

$$U = -\vec{p}\cdot\vec{E}$$

$$U = -\int_{90^\circ}^{\theta} \tau\, d\theta' = -\int_{90^\circ}^{\theta} pE\sin\theta\, d\theta'$$

$$U = -pE\int_{90^\circ}^{\theta} \sin\theta\, d\theta' = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$





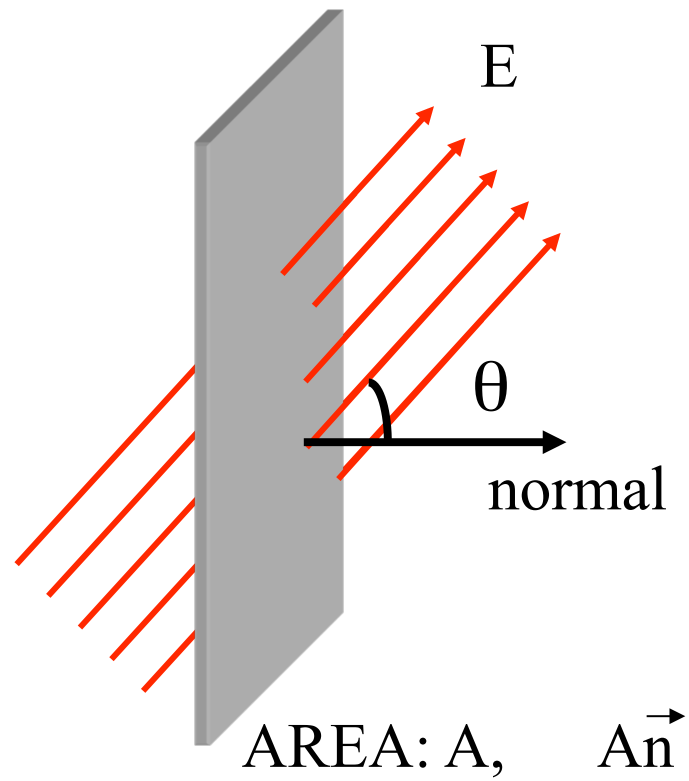
At point A ($\theta = 0$), U has a minimum value $U_{\min} = -pE$.
It is a position of stable equilibrium.

At point B ($\theta = 180^\circ$), U has a maximum value $U_{\max} = +pE$.
It is a position of unstable equilibrium.

Electric Flux: Planar Surface

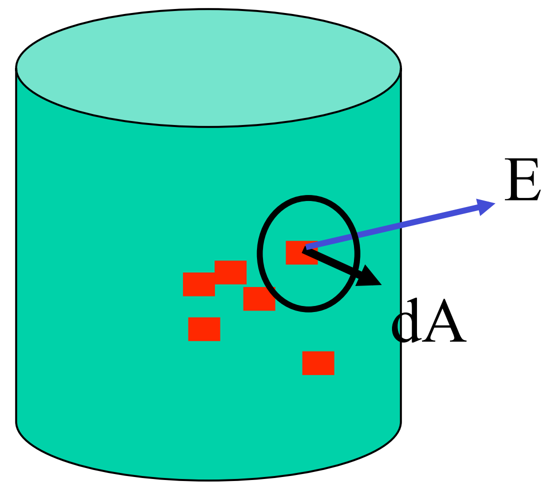
- Given:
  - planar surface, area A
  - uniform field E
  - E makes angle θ with NORMAL to plane
- Electric Flux:

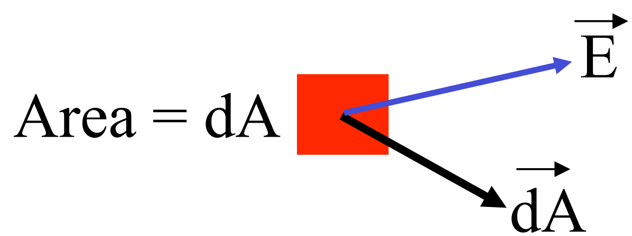
  $\Phi = \vec{\mathbf{E}} \bullet \vec{\mathbf{A}} = E\ A\ \cos\theta$
- Units: Nm^2/C
- Visualize: "Flow of Wind" Through "Window"



Electric Flux: General Surface

- For any general surface: break up into infinitesimal planar patches
- Electric Flux $\mathbf{\Phi = \int \vec{E} \bullet d\vec{A}}$
- Surface integral
- $\mathbf{d\vec{A}}$ is a vector normal to each patch and has a magnitude = $|\mathbf{d\vec{A}}|$=dA
- **CLOSED** surfaces:
  - define the vector dA as pointing **OUTWARDS**
  - Inward E gives negative flux Φ
  - Outward E gives positive flux Φ





Electric Flux: Example

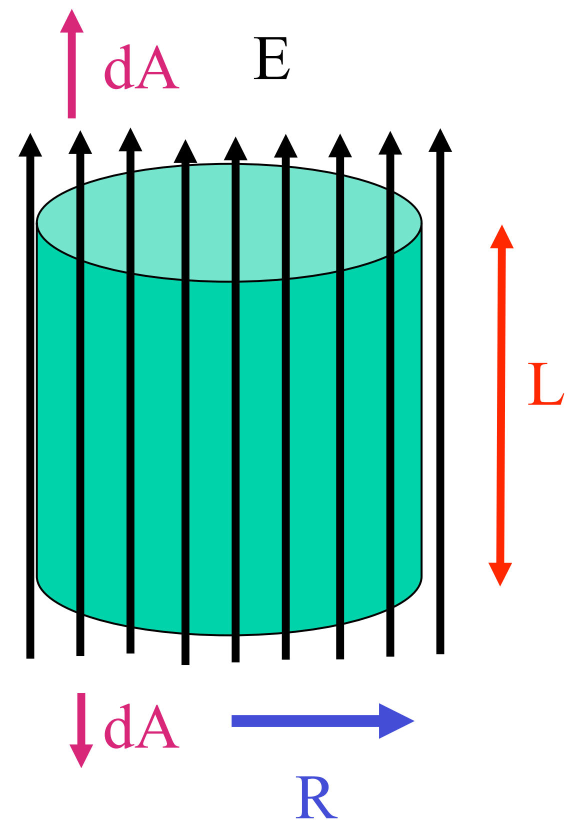
- Closed cylinder of length L, radius R
- Uniform E parallel to cylinder axis
- What is the total electric flux through surface of cylinder?

(a) $(2\pi RL)E$

(b) $2(\pi R^2)E$

(c) Zero $(\pi R^2)E-(\pi R^2)E=0$

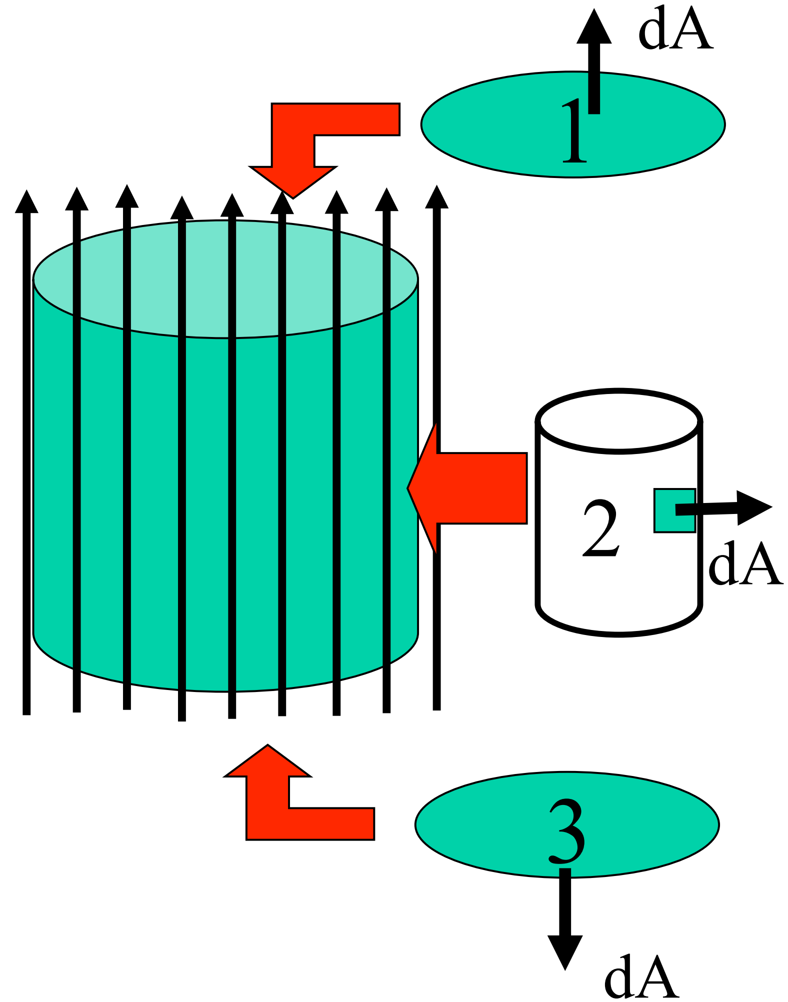
What goes in —
MUST come out!



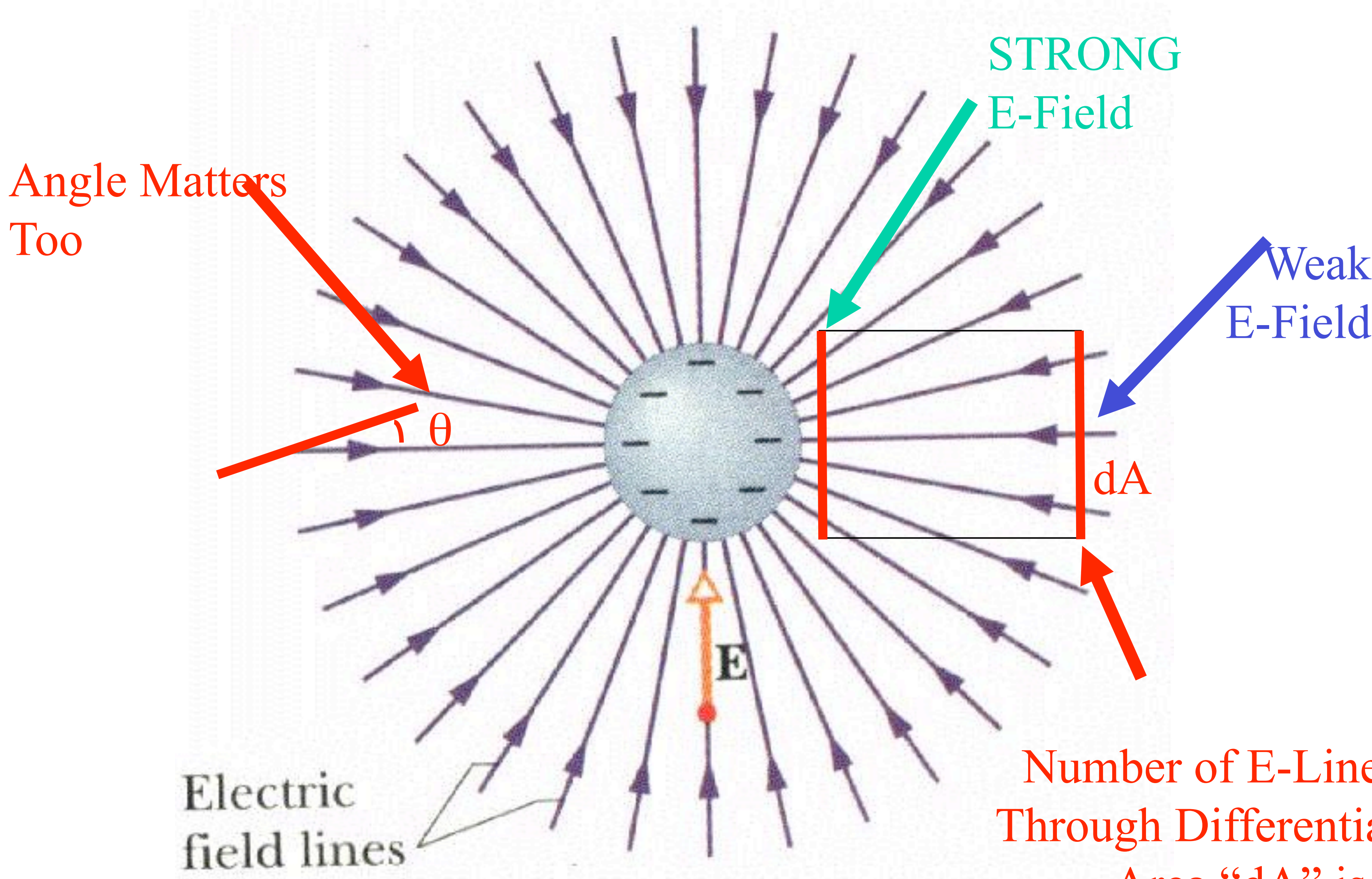
Hint!
Surface area of sides of cylinder: $2\pi RL$
Surface area of top and bottom caps (each): πR^2

Electric Flux: Example

- Note that E is **normal** to both bottom and top cap
- E is **parallel** to curved surface everywhere
- So: $\Phi = \Phi_1 + \Phi_2 + \Phi_3$ $= \pi R^2 E + 0 - \pi R^2 E$ $= 0!$
- Physical interpretation: total "inflow" = total "outflow"!

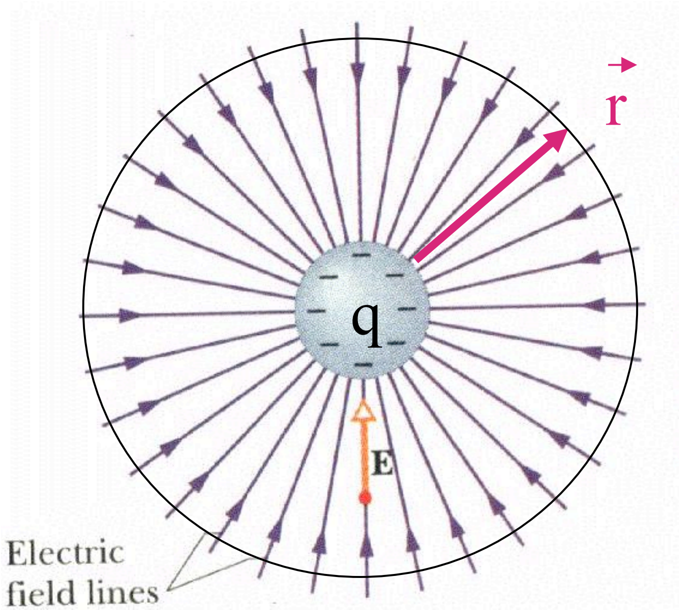


What? — The Flux!



Number of E-Lines Through Differential Area “dA” is a Measure of Strength

Electric Flux: Example



$$\vec{\mathbf{E}} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$ (Inward!)

$$d\vec{\mathbf{A}} = +(dA)\hat{\mathbf{r}}$$ (Outward!)

$$\vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = EdA\cos(180^\circ) = -EdA$$

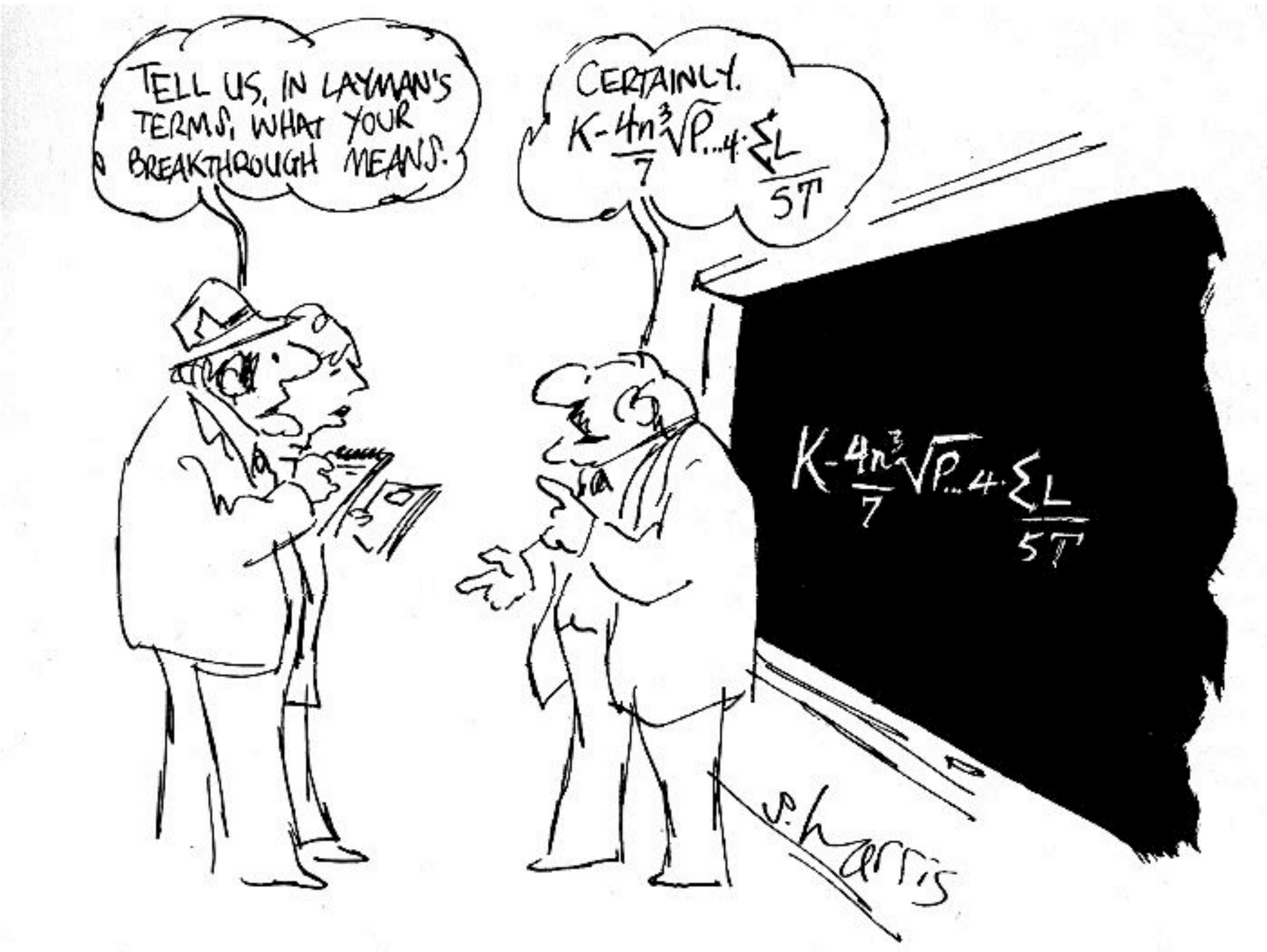
Since r is Constant on the Sphere — Remove E Outside the Integral!

$$\Phi = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = -E\oint dA = \left(-\frac{kq}{r^2}\right)(4\pi r^2)$$ Surface Area Sphere

$$= -\frac{q}{4\pi\varepsilon_0}(4\pi) = -q/\varepsilon_0$$ Gauss' Law: Special Case!

Summary

- An **electric dipole** in an electric field rotates to align itself with the field
- **Torque:** $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$
- Work done by electric field is negative of work by an **external agent**: $W_a = \Delta U = -W$
- **Electric flux** is $\boldsymbol{\Phi} = \int \mathbf{E} \cdot d\mathbf{A}$
- The area vector always points outward


TELL US, IN LAYMAN'S TERMS, WHAT YOUR BREAKTHROUGH MEANS.
CERTAINLY. $K - \frac{4n^3}{7}\sqrt{P} \ldots 4 \cdot \frac{\Sigma L}{5T}$
$K - \frac{4n^3}{7}\sqrt{P} \ldots 4 \cdot \frac{\Sigma L}{5T}$
S. Harris