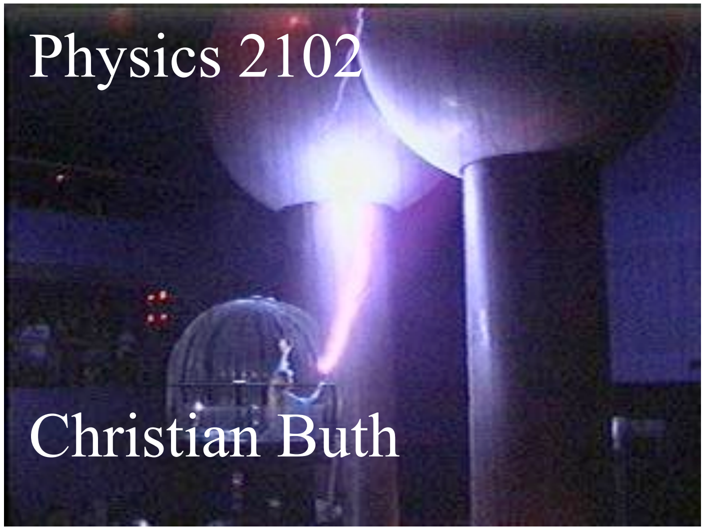
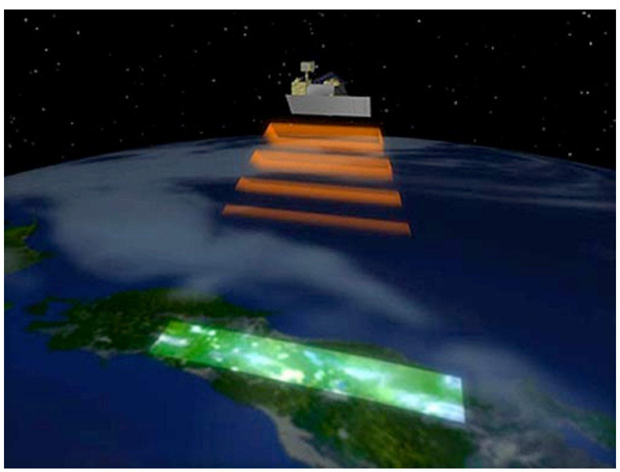
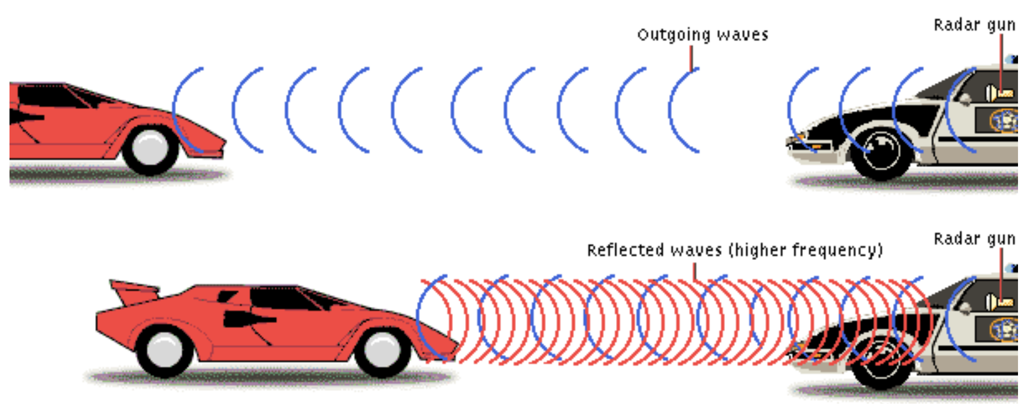


Lecture 41

Diffraction 2

04/29/2009





Review

- Interference only for coherent light, i.e., with a **phase relationship** that is time independent
- **Intensity** in double-slit interference:

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi$$

$$\phi = \frac{2\pi d}{\lambda} \sin\theta$$

- Use Huygens' Principle to find positions of **diffraction minima** of a single slit by subdividing the aperture

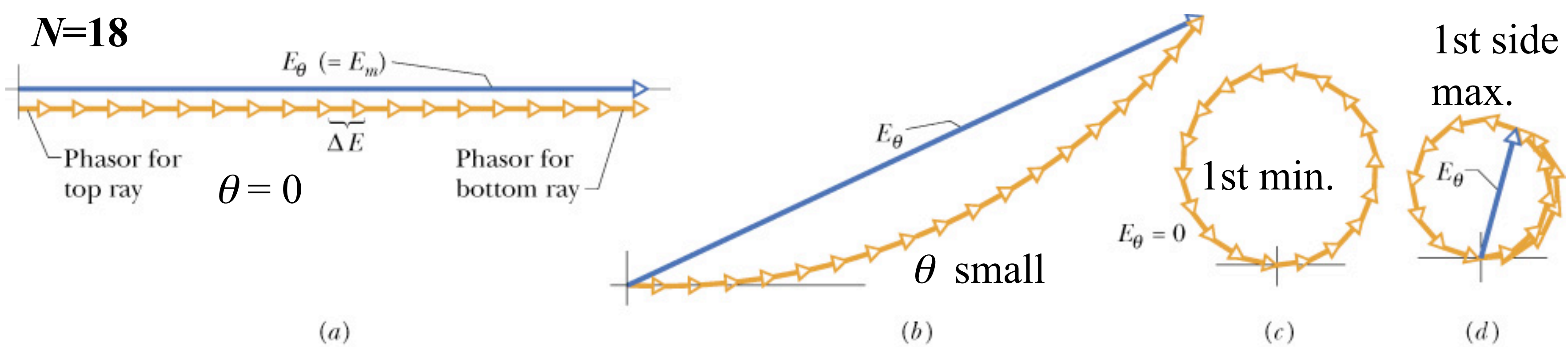
$$a \sin\theta = m\lambda, \quad \text{for } m = 1, 2, 3\ldots \text{ (minima-dark fringes)}$$

Intensity in Single-Slit Diffraction, Qualitatively

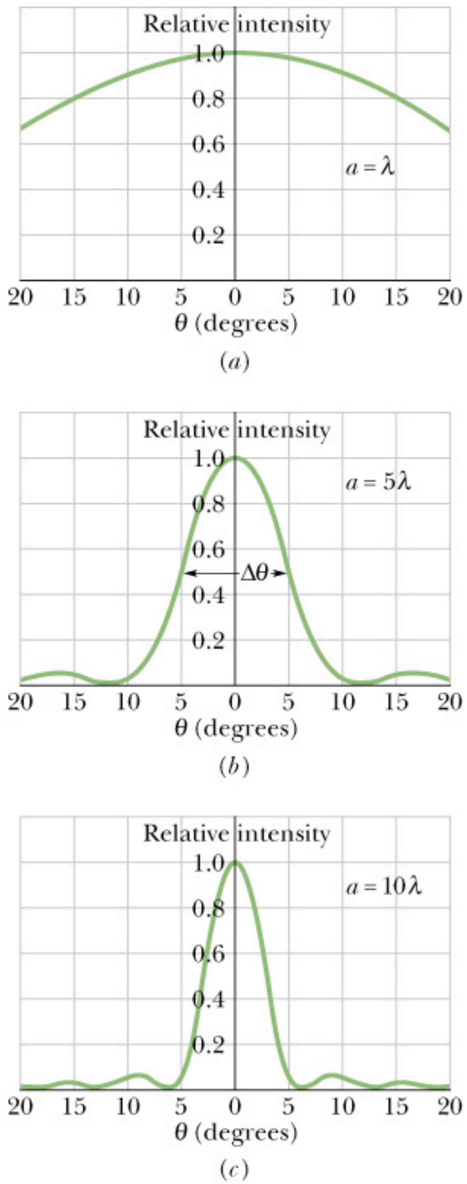
To obtain the locations of the minima, the slit was equally divided into N zones, each with width Δx. Each zone acts as a source of Huygens wavelets. Now these zones can be superimposed at the screen to obtain the intensity as a function of θ, the angle to the central axis.

To find the net electric field E_θ (intensity α E_θ^2) at point P on the screen, we need the phase relationships among the wavelets arriving from different zones:

$$\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix} = \left(\frac{2\pi}{\lambda}\right)\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix} \qquad \Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x \sin\theta)$$



Intensity in Single-Slit Diffraction, Quantitatively



Here we will show that the intensity at the screen due to a single slit is:

$$I(\theta) = I_m \left(\frac{\sin\alpha}{\alpha} \right)^2 \qquad (36\text{-}5)$$

$$\text{where} \quad \alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda}\sin\theta \qquad (36\text{-}6)$$

In Eq. 36-5, minima occur when:

$$\alpha = m\pi, \qquad \text{for } m = 1, 2, 3\ldots$$

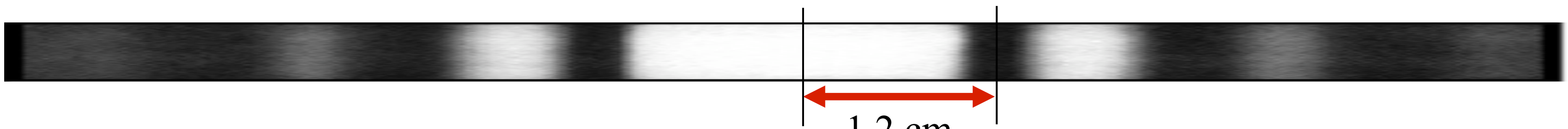
If we put this into Eq. 36-6 we find:

$$m\pi = \frac{\pi a}{\lambda}\sin\theta, \qquad \text{for } m = 1, 2, 3\ldots$$

$$\text{or} \qquad a\sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3\ldots$$

(minima-dark fringes)

Diffraction of a laser through a slit



Light from a helium-neon laser (λ = 633 nm) passes through a narrow slit and is seen on a screen 2.0 m behind the slit. The first minimum of the diffraction pattern is observed to be located 1.2 cm from the central maximum.

How wide is the slit?

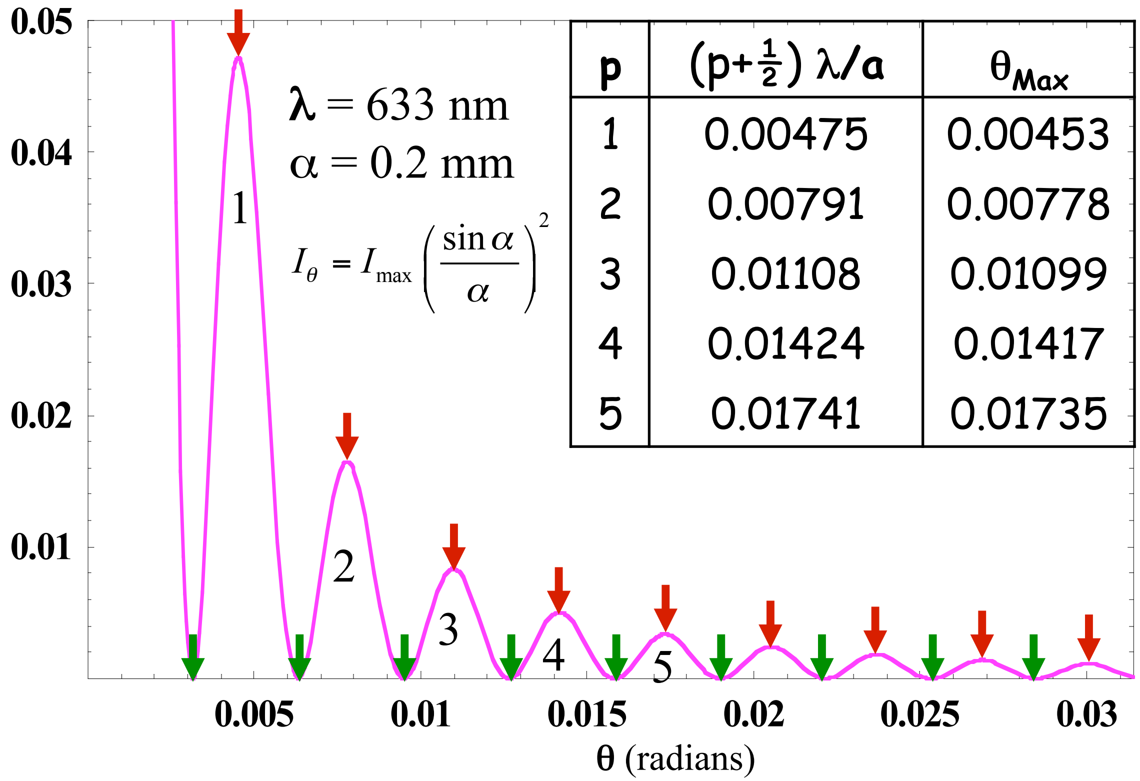
$$\theta_1 = \frac{y_1}{L} = \frac{(0.012\ \text{m})}{(2.00\ \text{m})} = 0.0060\ \text{rad}$$

$$a = \frac{\lambda}{\sin\theta_1} \cong \frac{\lambda}{\theta_1} = \frac{(6.33\times10^{-7}\ \text{m})}{(6.00\times10^{-3}\ \text{rad})} = 1.06\times10^{-4}\ \text{m} = 0.106\ \text{mm}$$

Angles of the Secondary Maxima

The diffraction minima are ***precisely*** at the angles where $\sin\theta = p\,\lambda/a$ and $\alpha = p\pi$ (so that $\sin\alpha = 0$).

However, the diffraction maxima are ***not quite*** at the angles where $\sin\theta = (p+\frac{1}{2})\,\lambda/a$ and $\alpha = (p+\frac{1}{2})\pi$ (so that $|\sin\alpha| = 1$).



p	$(p+\frac{1}{2})\,\lambda/a$	θ_{Max}
1	0.00475	0.00453
2	0.00791	0.00778
3	0.01108	0.01099
4	0.01424	0.01417
5	0.01741	0.01735

To find the maxima, one must look ***near*** $\sin\theta = (p+\frac{1}{2})\,\lambda/a$, for places where the slope of the diffraction pattern goes to zero, i.e., where $d[(\sin\alpha/\alpha)^2]/d\theta = 0$. This is a transcendental equation that must be solved numerically. The table gives the θ_{Max} solutions. Note that $\theta_{Max} < (p+\frac{1}{2})\,\lambda/a$.

Diffraction by a Circular Aperture

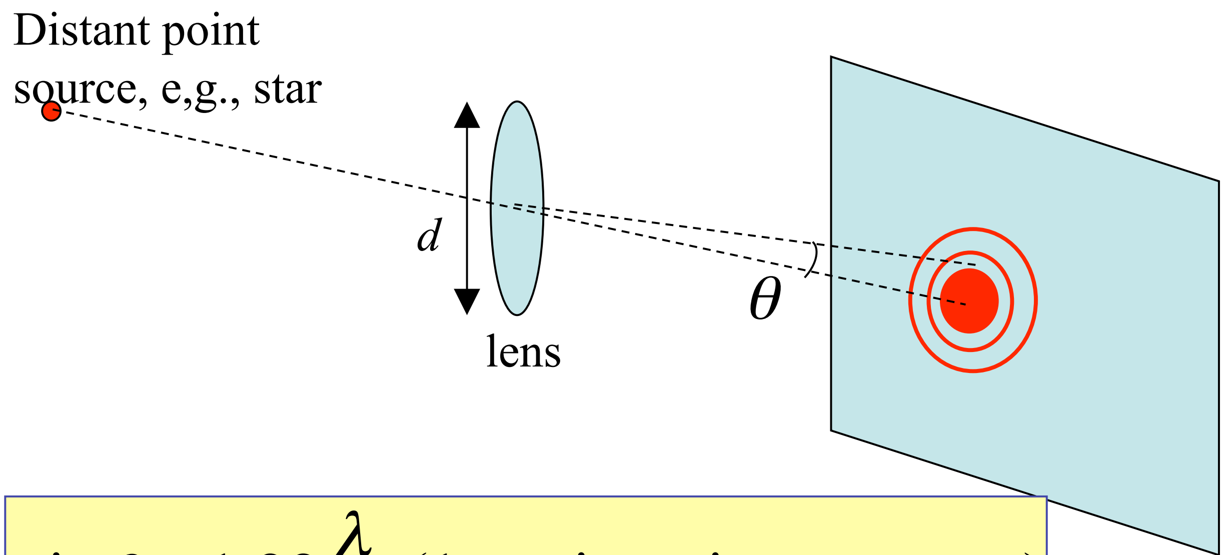
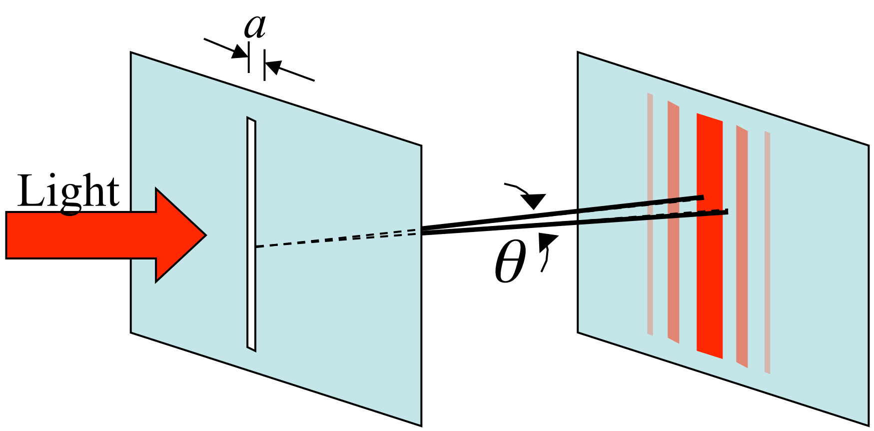
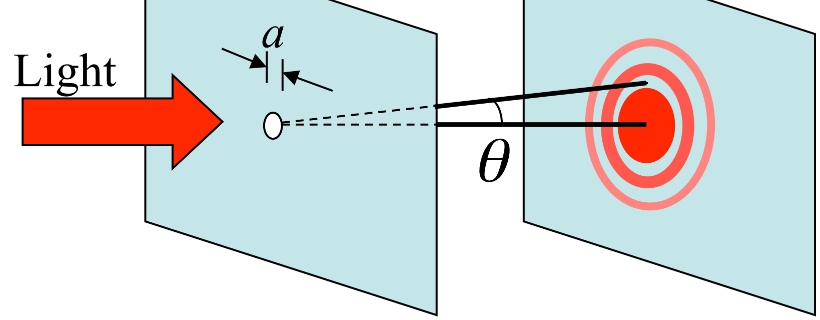


Image is not a point, as expected from geometrical optics! Diffraction is responsible for this image pattern.

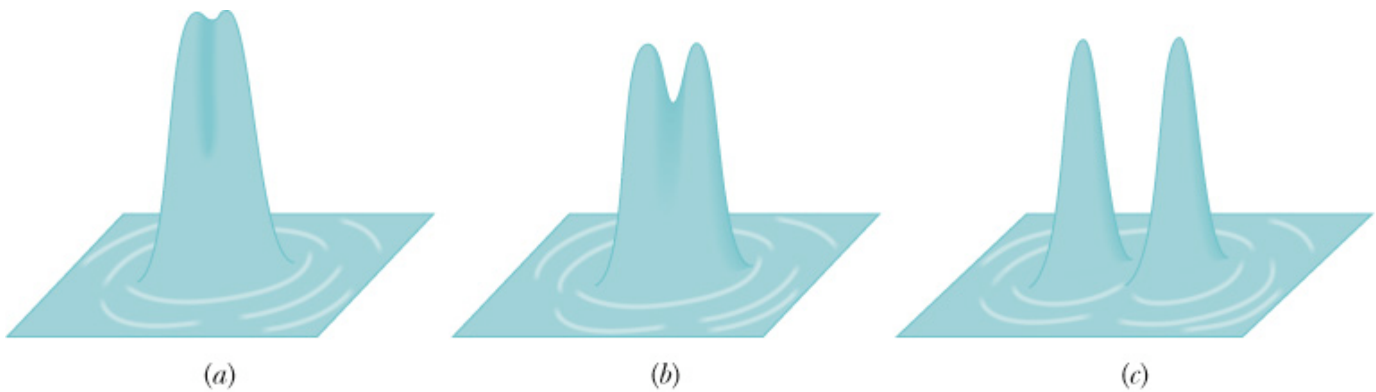
$$\sin\theta = 1.22\frac{\lambda}{d} \quad \text{(1st min.- circ. aperture)}$$





Resolvability

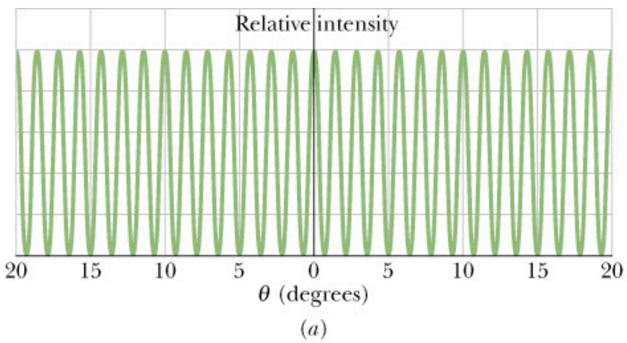
Rayleigh's Criterion: Two point sources are barely resolvable if their angular separation θ_R results in the central maximum of the diffraction pattern of one source's image centered on the first minimum of the diffraction pattern of the other source's image



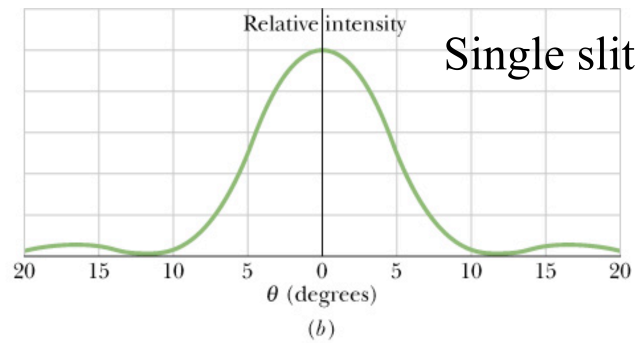
$$\theta_R = \sin^{-1}\left(1.22\frac{\lambda}{d}\right) \overset{\theta_R \text{ small}}{\approx} 1.22\frac{\lambda}{d} \quad \text{(Rayleigh's criterion)}$$

Diffraction by a Double Slit

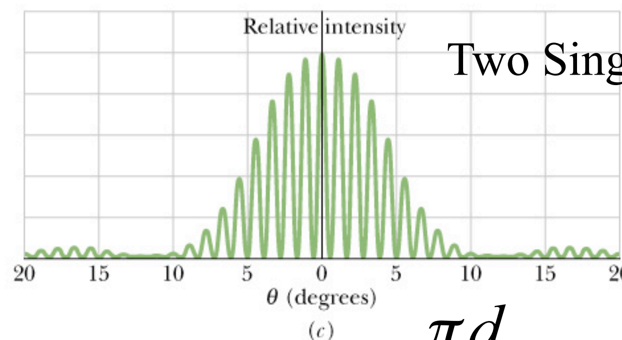
In the double-slit experiment, we assumed that the slit width $a \ll \lambda$. What if this is not the case?



Two vanishingly narrow slits $a \ll \lambda$



Single slit $a \sim \lambda$



Two Single slits $a \sim \lambda$

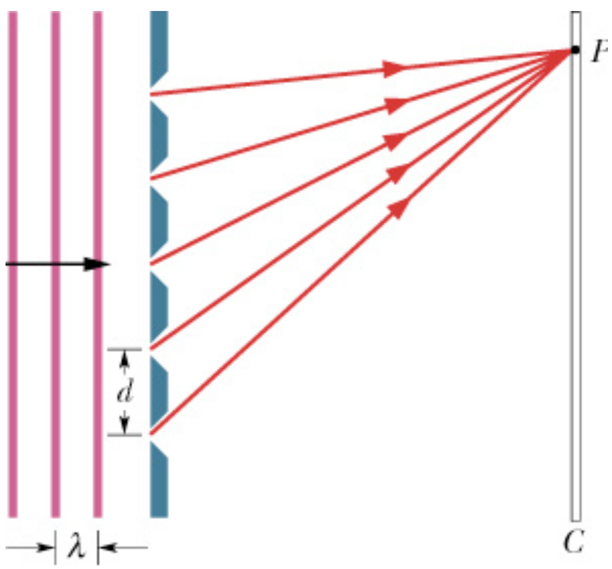
$$I(\theta) = I_m \left(\cos^2 \beta\right)\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{(double slit)}$$

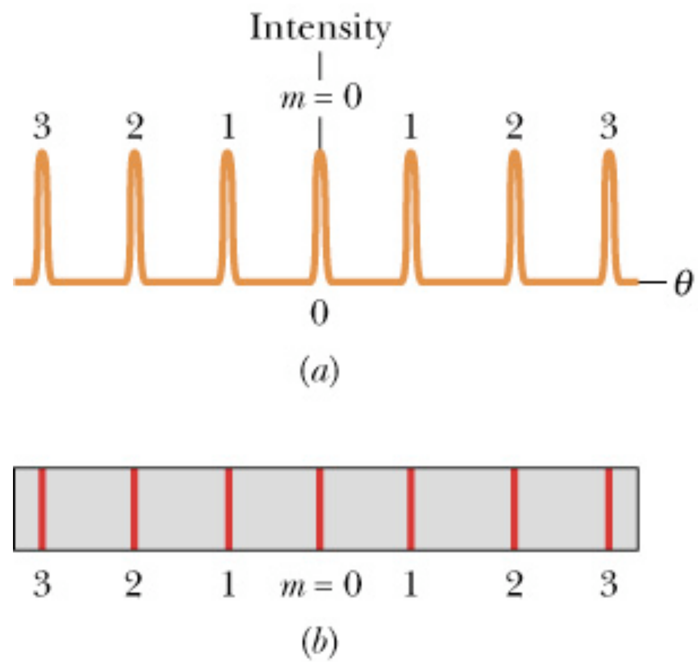
$$\beta = \frac{\pi d}{\lambda}\sin\theta$$

$$\alpha = \frac{\pi a}{\lambda}\sin\theta$$

Diffraction Gratings

A device with *N* slits (rulings) can be used to manipulate light, such as separate different wavelengths of light that are contained in a single beam. How does a diffraction grating affect monochromatic light?

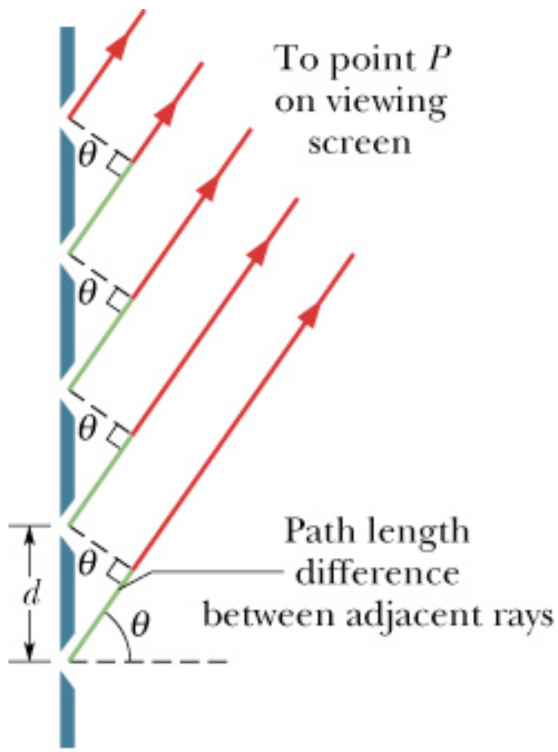


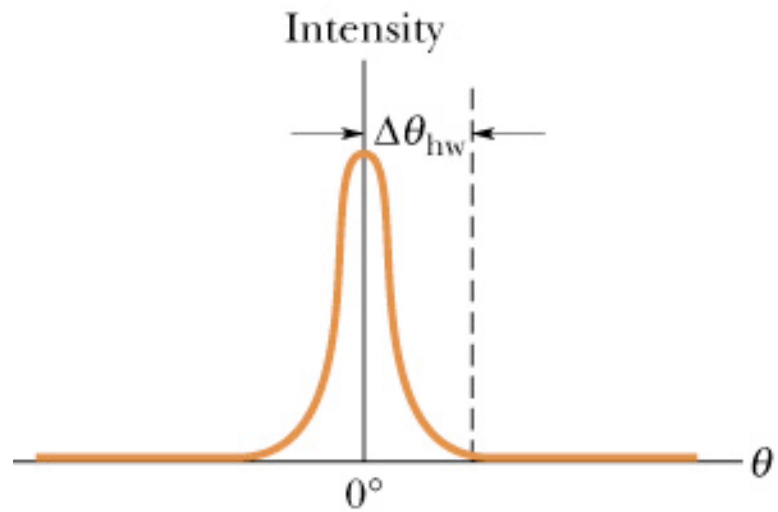


$$d \sin\theta = m\lambda \quad \text{for } m = 0,1,2\ldots \quad \text{(maxima-lines)}$$

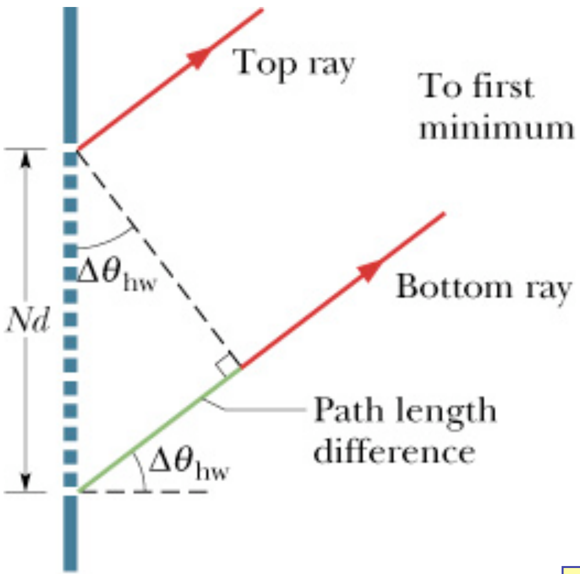
Width of Lines

The ability of the diffraction grating to resolve (separate) different wavelengths depends on the width of the lines (maxima).





Width of Lines, cont'd



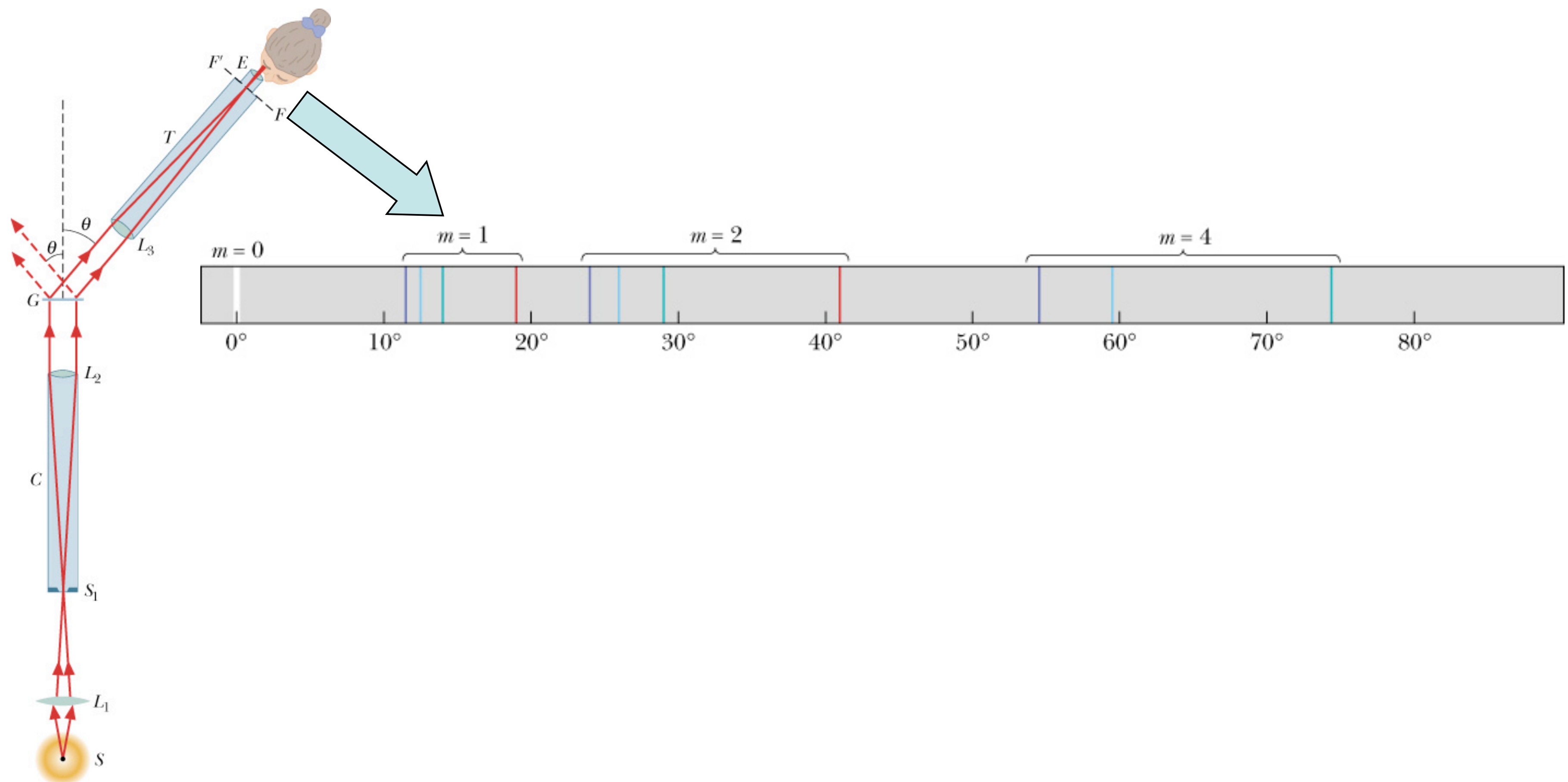
$$Nd\sin\Delta\theta_{\mathrm{hw}} = \lambda, \quad \sin\Delta\theta_{hw} \approx \Delta\theta_{hw}$$

$$\Delta\theta_{\mathrm{hw}} = \frac{\lambda}{Nd} \quad \text{(half width of central line)}$$

$$\Delta\theta_{\mathrm{hw}} = \frac{\lambda}{Nd\cos\theta} \quad \text{(half width of line at } \theta\text{)}$$

Grating Spectroscope

Separates different wavelengths (colors) of light into distinct diffraction lines



Summary

- To predict the interference pattern of a multi-slit system, we must combine interference and diffraction effects.
- **Rayleigh's Criterion** for separability of two points
- Intensity in single-slit diffraction:

$$I(\theta) = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\text{where } \alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda}\sin\theta \qquad (36\text{-}6)$$

- Double-slit diffraction:

$$I(\theta) = I_m \left(\cos^2\beta\right)\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{(double slit)}$$