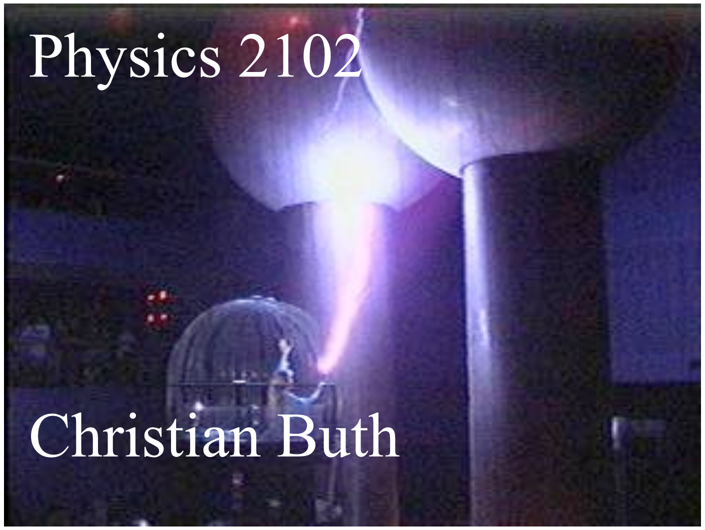
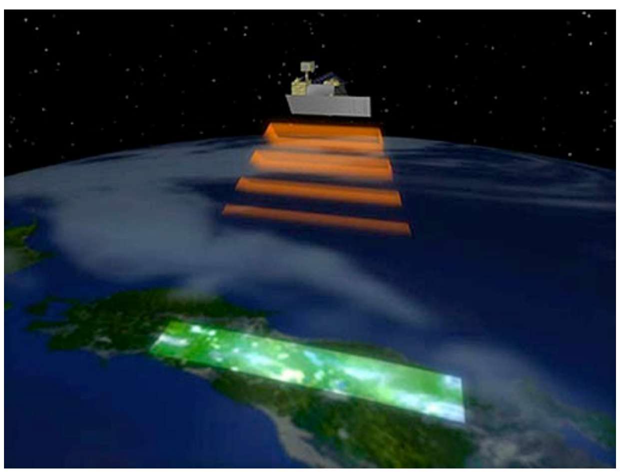
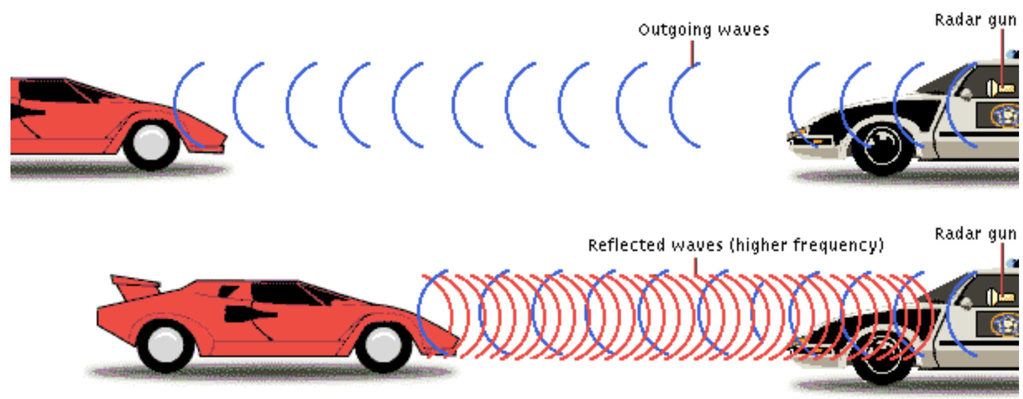


Lecture 40

Diffraction 1

04/27/2009





Review 1

- **Huygen's principle**: All points in a wavefront serve as point sources of spherical secondary waves
- The frequency of light in a medium is the same as it is in vacuum

Index of Refraction: $n = \frac{c}{v}$

- Wavelength changes

$$\lambda_n = \lambda \frac{v_n}{c} = \frac{\lambda}{n}$$

Review 2

- **Diffraction** of light occurs at openings of the order of the wave length of the light
- **Double slit experiment**:

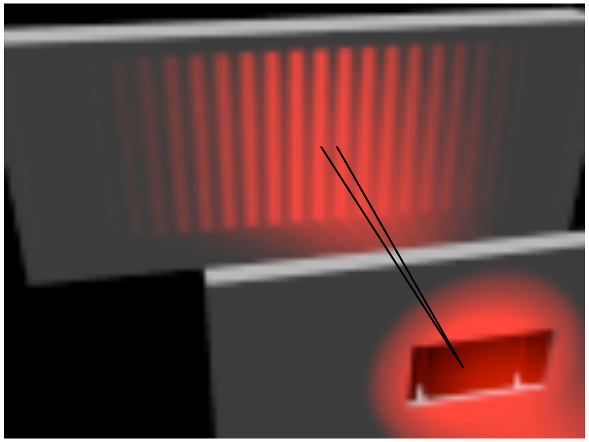
Maxima-bright fringes:

$$d\sin\theta = m\lambda \quad \text{for } m = 0, 1, 2, \ldots$$

Minima-dark fringes: $d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \quad \text{for } m = 0, 1, 2, \ldots$

Example

Red laser light (λ=633nm) goes through two slits 1cm apart, and produces a diffraction pattern on a screen 55cm away. How far apart are the fringes near the center?



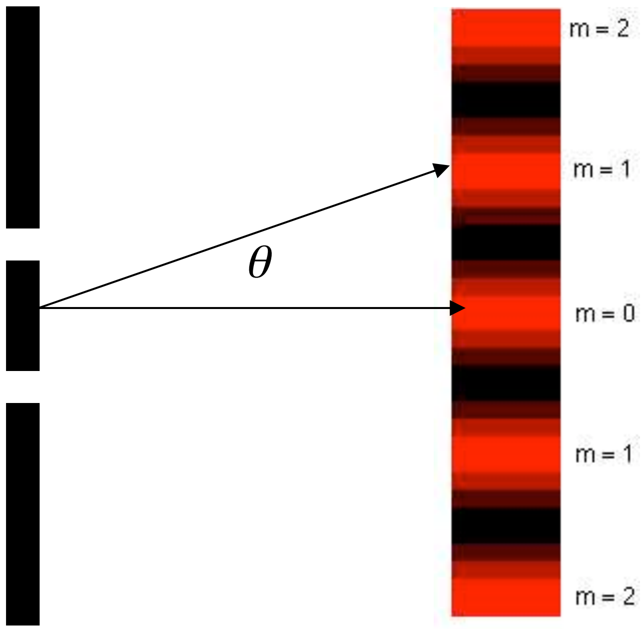
If the fringes are near the center, we can use
$\sin\theta \sim \theta$, and then
$m\lambda = d\sin\theta \sim d\theta \Rightarrow \theta = m\lambda/d$ is the angle for each maximum (in radians)
$\Delta\theta = \lambda/d$ =is the "angular separation".
The distance between the fringes is then
$\Delta x = L\Delta\theta = L\lambda/d = 55\text{cm}\ 633\text{nm}/1\text{cm} = 35\ \mu\text{m}$

For the spacing to be 1mm, we need $d \sim L\lambda/1\text{mm} = 0.35\text{mm}$

Example

In a double slit experiment, we can measure the wavelength of the light if we know the distances between the slits and the angular separation of the fringes.
If the separation between the slits is 0.5mm and the first order maximum of the interference pattern is at an angle of 0.059^o from the center of the pattern, what is the wavelength and color of the light used?

$d \sin\theta = m\lambda$ =>
$\lambda = 0.5\text{mm} \sin(0.059^o)$
$= 5.15 \times 10^{-7}\text{m} = 515\text{nm}$ ~ green



Coherence

Two sources can produce an interference that is stable over time, if their light has a **phase relationship** that does not change with time: $E(t)=E_0\cos(\omega t+\phi)$.

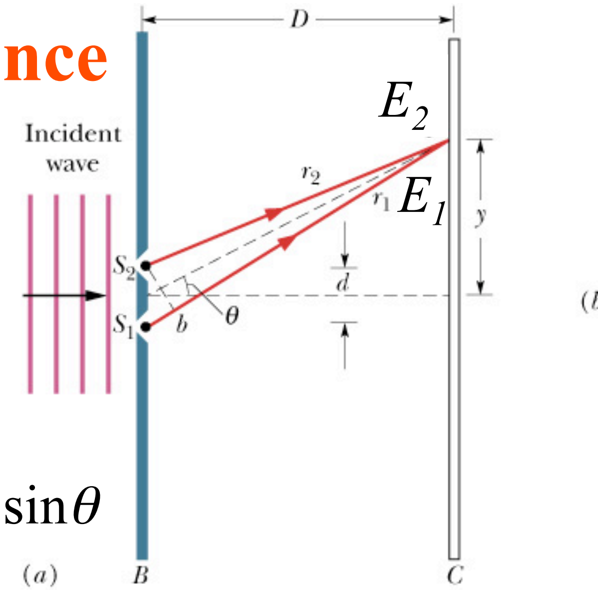
Coherent sources: Phase ϕ must be well defined and constant

Sunlight is coherent over a short length and time range

Since laser light is produced by cooperative behavior of atoms, it is coherent of long length and time ranges

Incoherent sources: ϕ jitters randomly in time, no stable interference occurs

Intensity in Double-Slit Interference



$$E_1 = E_0 \sin \omega t \quad \text{and} \quad E_2 = E_0 \sin(\omega t + \phi)$$

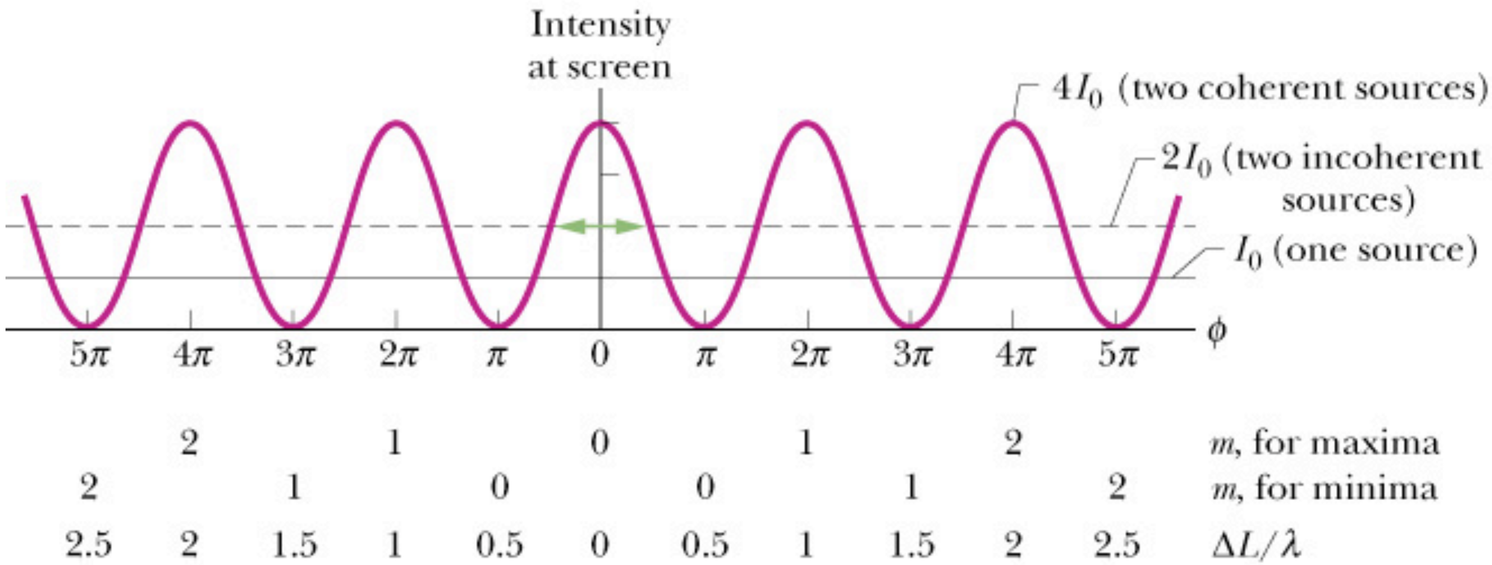
$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi$$

$$\phi = \frac{2\pi d}{\lambda} \sin \theta$$

Maxima when: $\frac{1}{2}\phi = m\pi \;$ for $m = 0, 1, 2, \ldots \rightarrow \phi = 2m\pi = \dfrac{2\pi d}{\lambda}\sin\theta$

$\rightarrow d \sin \theta = m\lambda \;$ for $m = 0, 1, 2, \ldots \quad$ (maxima)

Minima when: $\frac{1}{2}\phi = \left(m + \frac{1}{2}\right)\pi \rightarrow d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \;$ for $m = 0, 1, 2, \ldots \quad$ (minima)



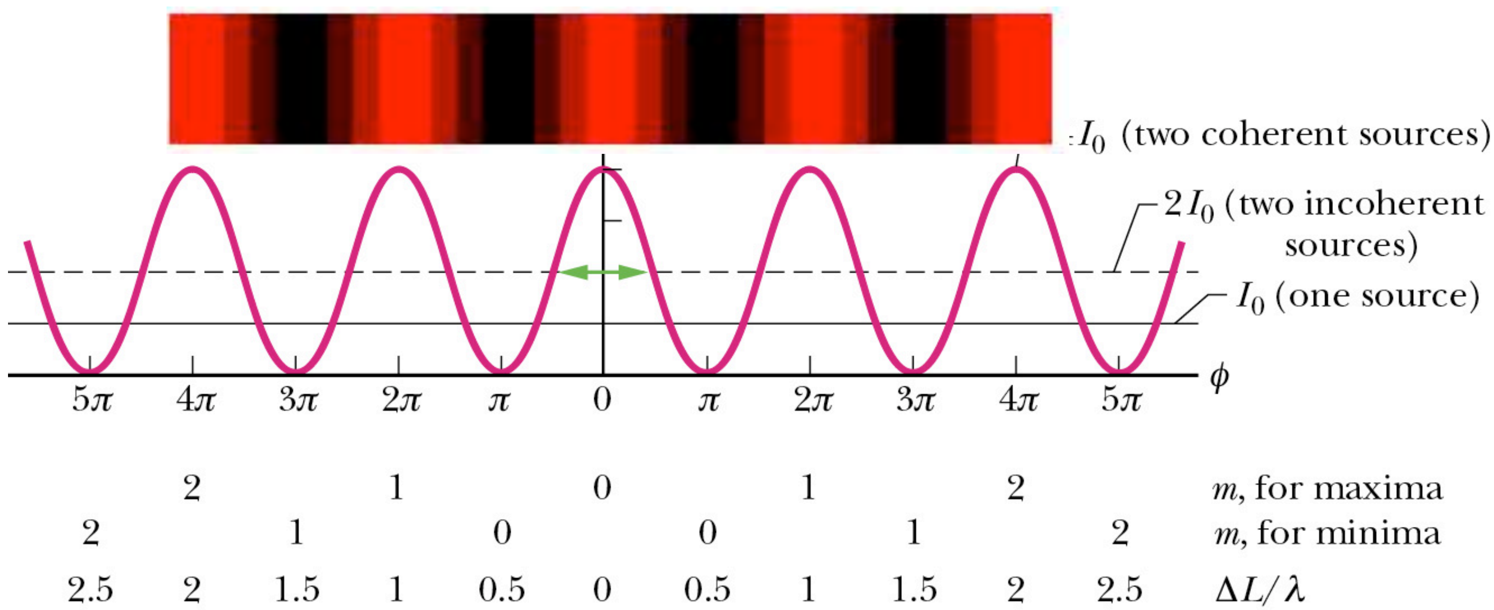
Example

A double slit experiment has a screen 120cm away from the slits, which are 0.25cm apart. The slits are illuminated with coherent 600nm light. At what distance above the central maximum is the average intensity on the screen 75% of the maximum?

$I/I_0 = 4\cos^2\phi/2$; $I/I_{max} = \cos^2\phi/2 = 0.75 \Rightarrow \phi = 2\cos^{-1}(0.75)^{1/2} = 60^o = \pi/3$ rad

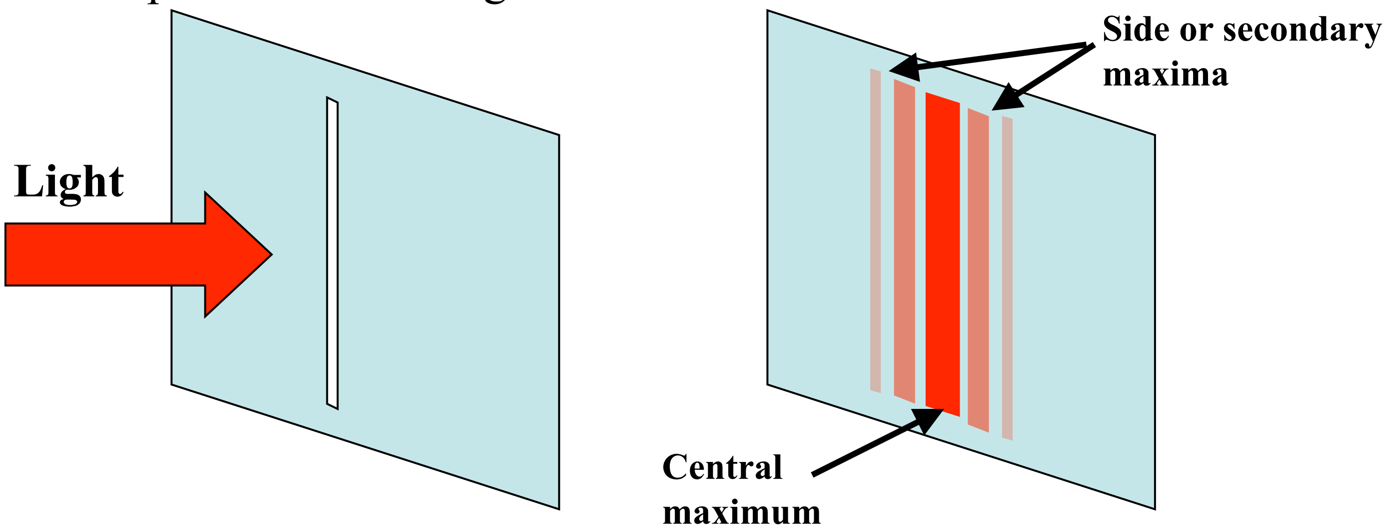
$\phi = (2\pi d/\lambda)\sin\theta \Rightarrow \theta = \sin^{-1}(\lambda/2\pi d)\phi = 0.0022^o = 40$ μrad (small!)

$y = L\theta = 48$μm



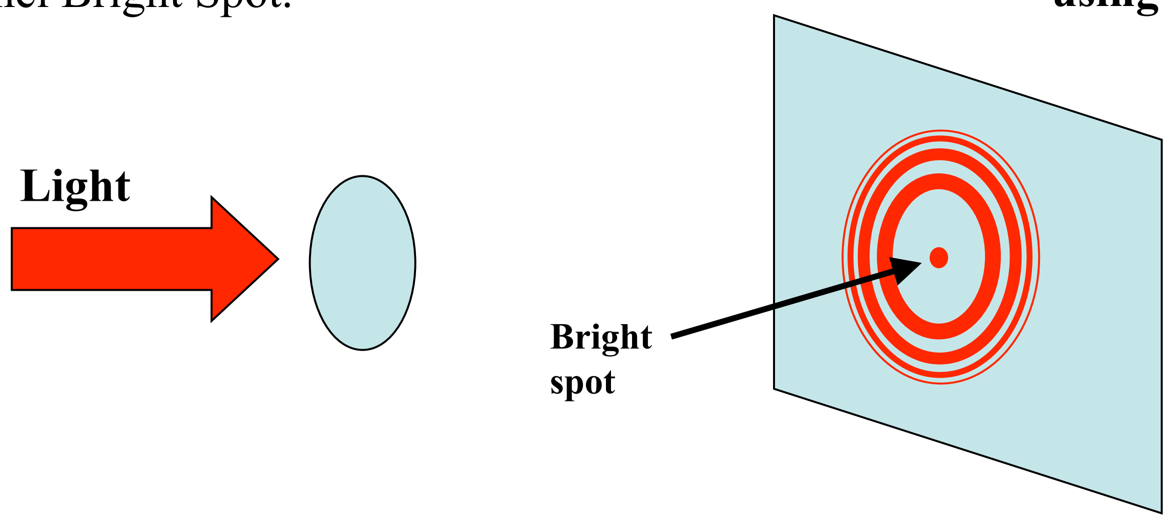
Diffraction and the Wave Theory of Light

Diffraction pattern from a single narrow slit.

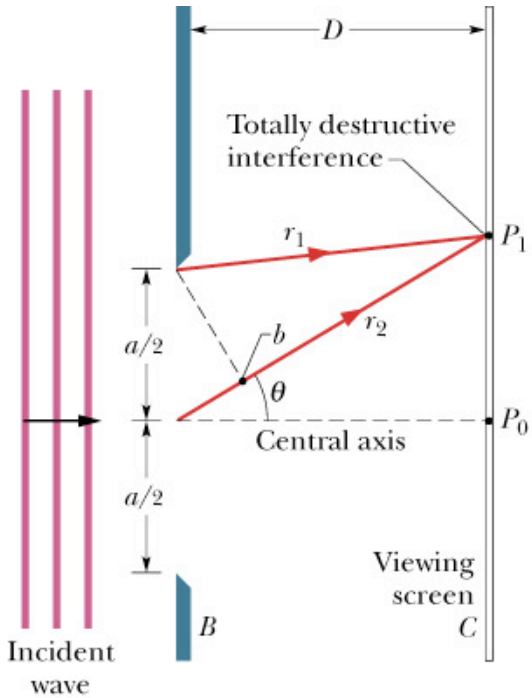


These patterns cannot be explained using geometrical optics!

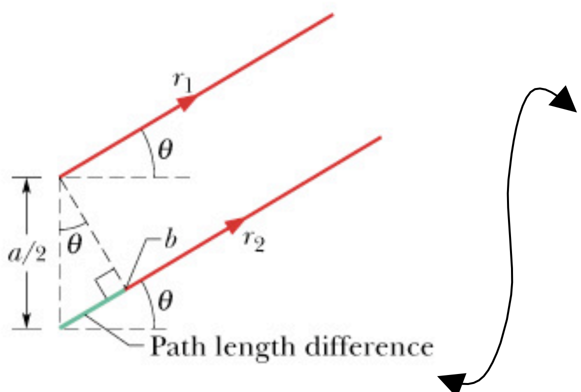
Fresnel Bright Spot.



Diffraction by a Single Slit: Locating the Minima



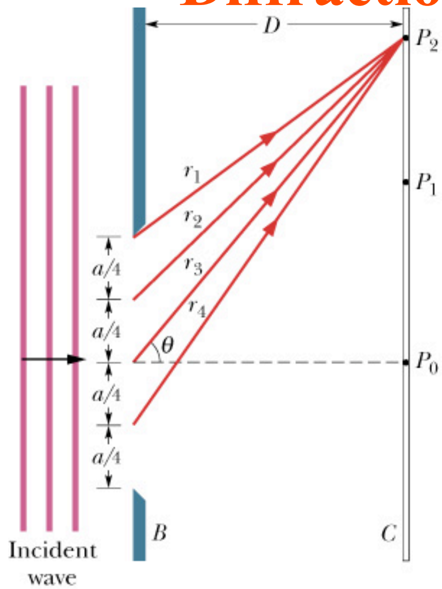
(a)



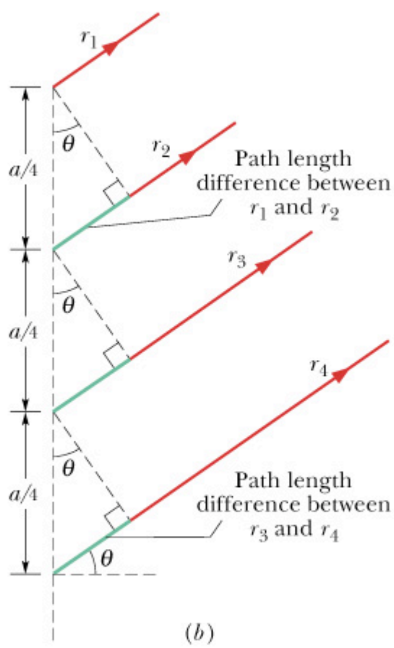
(b)

- **Path length difference** between rays r_1 and r_2 is $\lambda/2$
- Two rays out of phase at P_1 resulting in **destructive interference**
- Path length difference is **distance** from starting point of r_2 at center of the slit to point b
- For $D>>a$, the path length difference between rays r_1 and r_2 is $(a/2)\sin\theta$

Diffraction by a Single Slit: Locating the Minima, cont'd



(a)



(b)

Repeat previous analysis for pairs of rays, each separated by a vertical distance of $a/2$ at the slit.

Setting path length difference to $\lambda/2$ for each pair of rays, we obtain the first dark fringes at:

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \rightarrow a\sin\theta = \lambda \qquad \text{(first minimum)}$$

For second minimum, divide slit into 4 zones of equal widths $a/4$ (separation between pairs of rays). Destructive interference occurs when the path length difference for each pair is $\lambda/2$.

$$\frac{a}{4}\sin\theta = \frac{\lambda}{2} \rightarrow a\sin\theta = 2\lambda \qquad \text{(second minimum)}$$

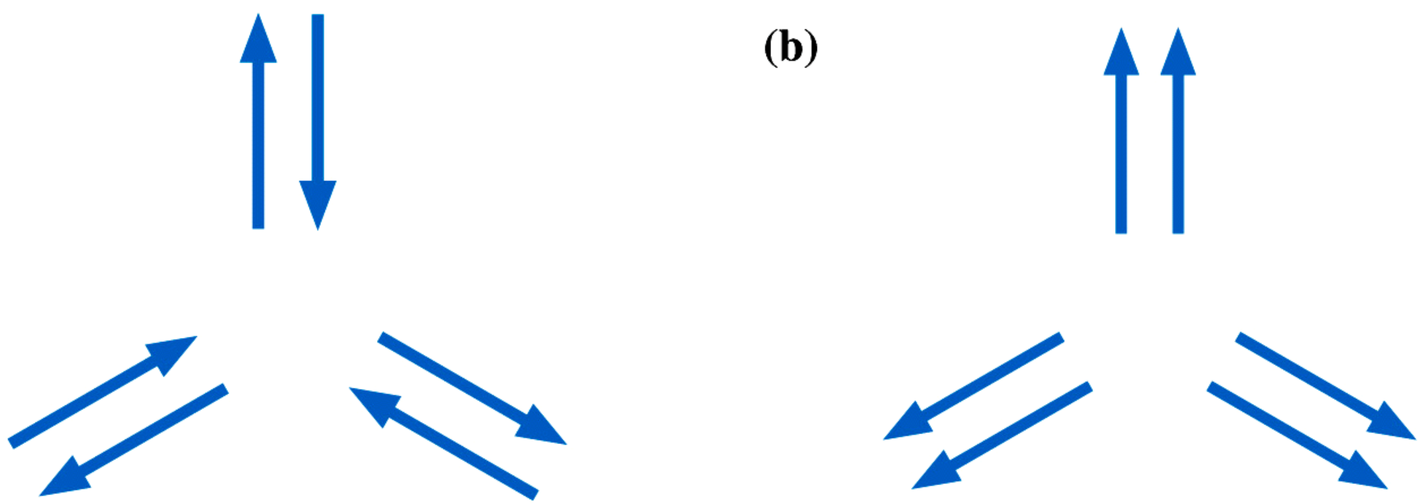
Dividing the slit into increasingly larger even numbers of zones, we can find higher order minima:

$$a\sin\theta = m\lambda, \quad \text{for } m = 1,2,3\ldots \qquad \text{(minima-dark fringes)}$$

Pairing and Interference

• Can the same technique be used to find the **maxima**, by choosing pairs of wavelets with path lengths that differ by λ?

• **No.** Pair-wise destructive interference works, but pair-wise constructive interference does **not necessarily** lead to maximum constructive interference. Below is an example demonstrating this



(a) Each pair of vectors interferes destructively. The vector sum of all six vectors is zero.

(b) Each pair of vectors interferes constructively. Even so, the vector sum of all six vectors is zero.

Summary

- Interference only for coherent light, i.e., with a **phase relationship** that is time independent
- **Intensity** in double-slit interference:

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi$$

$$\phi = \frac{2\pi d}{\lambda} \sin\theta$$

- Use Huygens' Principle to find positions of **diffraction minima** of a single slit by subdividing the aperture

$$a \sin\theta = m\lambda, \quad \text{for } m = 1, 2, 3\ldots \quad \text{(minima-dark fringes)}$$