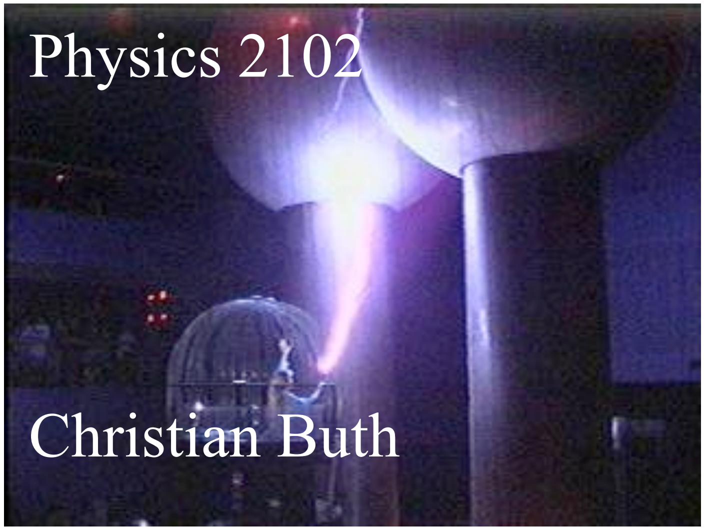


Lecture 34

Electromagnetic Waves 2

04/13/2009

Review

- Maxwell's laws: **electromagnetic (EM) waves** (in vacuum)
- EM waves travel at the **speed of light**, are transversal
- ***E***, ***B*** are **perpendicular**; form right-handed coordinate system with propagation direction and vary sinusoidally
- **Poynting Vector *S***: energy in propagation direction

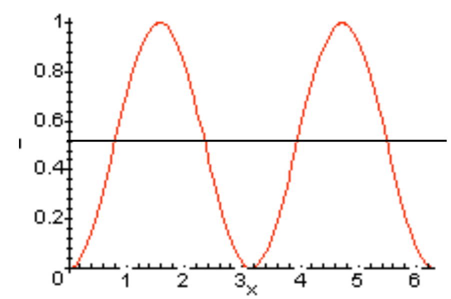
$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

EM Wave Intensity, Energy Density

A better measure of the amount of energy in an EM wave is obtained by **averaging** the Poynting vector over one wave cycle. The resulting quantity is called **intensity**. Units are also Watt / m^2

$$I = \bar{S} = \frac{1}{c\mu_0}\overline{E^2} = \frac{1}{c\mu_0}E_m{}^2\,\overline{\sin^2(kx-\omega t)}$$

The average of sin^2 over one cycle is ½:



$$I = \frac{1}{2c\mu_0}E_m{}^2 \quad \text{or,} \quad I = \frac{1}{c\mu_0}E_{rms}{}^2$$

Both fields have the same energy density

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (cB)^2 = \frac{1}{2}\varepsilon_0\frac{B^2}{\varepsilon_0\mu_0} = \frac{1}{2}\frac{B^2}{\mu_0} = u_B$$

The total EM energy density is then

$$u = \varepsilon_0 E^2 = B^2/\mu_0$$

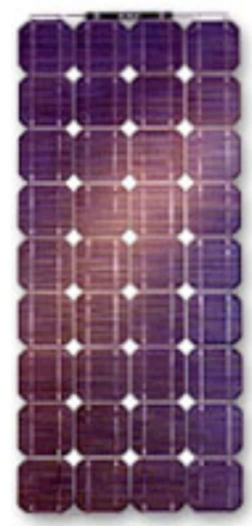
Solar Energy

The light from the sun has an intensity of about $1 kW/m^2$. What would be the total power incident on a roof of dimensions 8x20m?

$I = 1 kW/m^2$ is power per unit area

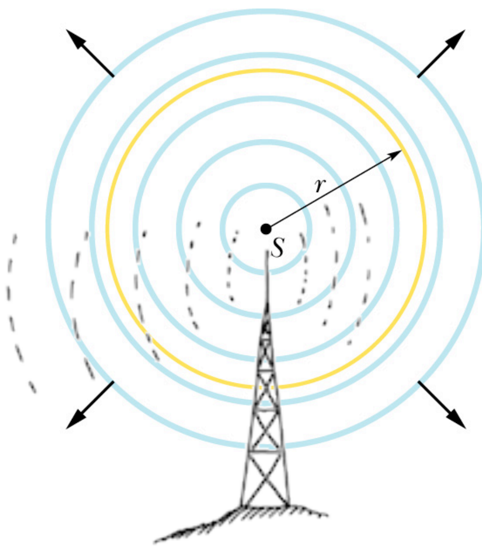
$P = IA = (10^3\ W/m^2) \times 8m \times 20m = 0.16\ MW$!!

The solar panel shown (BP-275) has dimensions 47in x 29in. The incident power is then 880 W. The actual solar panel delivers 75W (4.45A at 17V): less than 10% efficiency…



EM spherical waves

The intensity of a wave is power *per unit area*. If one has a source that emits isotropically (equally in all directions) the power emitted by the source pierces a larger and larger sphere as the wave travels outwards



$$I = \frac{P_s}{4\pi r^2}$$

So the power per unit area decreases as the inverse of distance squared

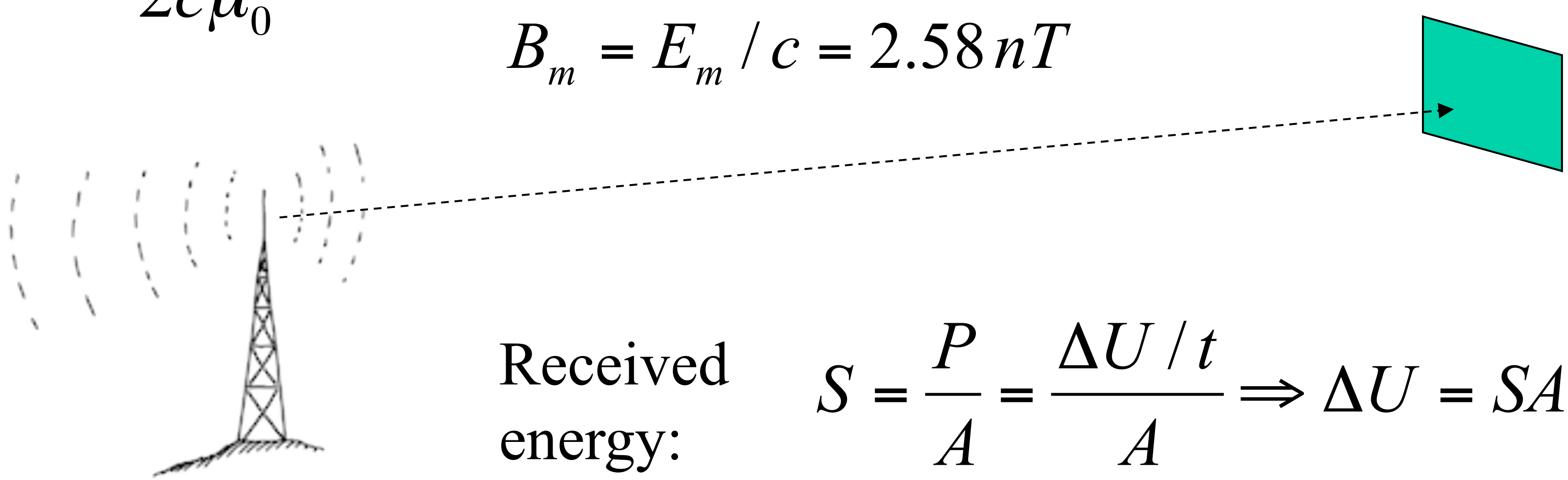
Example

A radio station transmits a 10 kW signal at a frequency of 100 MHz. (We will assume it radiates as a point source). At a distance of 1km from the antenna, find the amplitude of the electric and magnetic field strengths, and the energy incident normally on a square plate of side 10cm in 5min

$$I = \frac{P_s}{4\pi r^2} = \frac{10kW}{4\pi(1km)^2} = 0.8mW/m^2$$

$$I = \frac{1}{2c\mu_0}E_m^{\ 2} \Rightarrow E_m = \sqrt{2c\mu_0 I} = 0.775V/m$$

$$B_m = E_m / c = 2.58\,nT$$

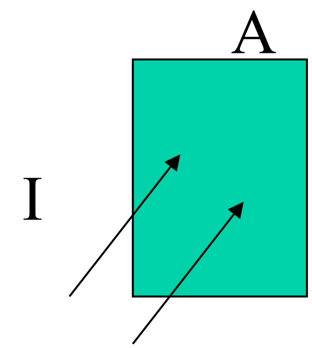


Received energy: $S = \frac{P}{A} = \frac{\Delta U / t}{A} \Rightarrow \Delta U = SAt = 2.4\,mJ$

Radiation Pressure

Waves not only carry energy but also momentum. The effect is very small (we don't ordinarily feel pressure from light). If light is completely absorbed during an interval Δt, the momentum transferred is given by $\Delta p = \frac{\Delta U}{c}$ and twice as much if reflected.

Newton's law: $F = \frac{\Delta p}{\Delta t}$



Supposing one has a wave that hits a surface of area A (perpendicularly), the amount of energy transferred to that surface in time Δt will be

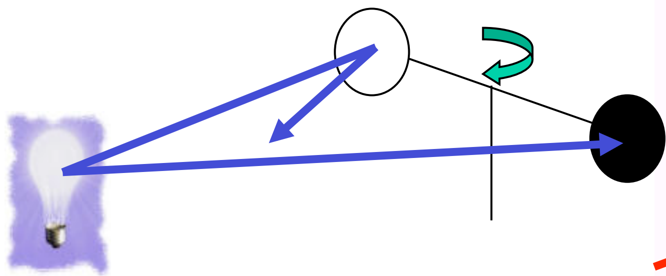
$$\Delta U = IA\Delta t \quad \text{therefore} \quad \Delta p = \frac{IA\Delta t}{c} \quad \longrightarrow \quad F = \frac{IA}{c}$$

Radiation pressure:

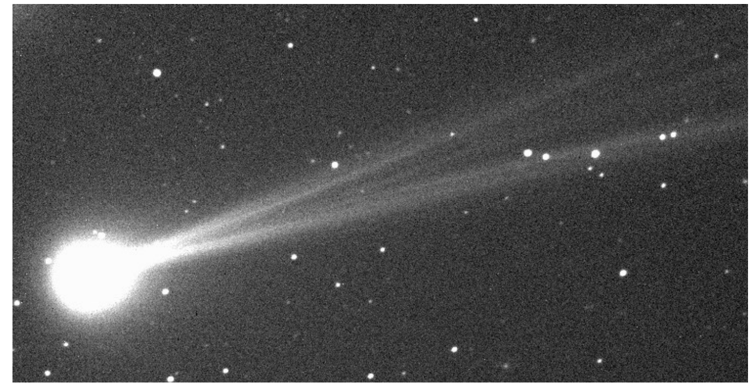
$$p_r = \frac{I}{c} \text{ (total absorption)}, \quad p_r = \frac{2I}{c} \text{ (total reflection)}$$

Radiation pressure: examples

Solar mills?

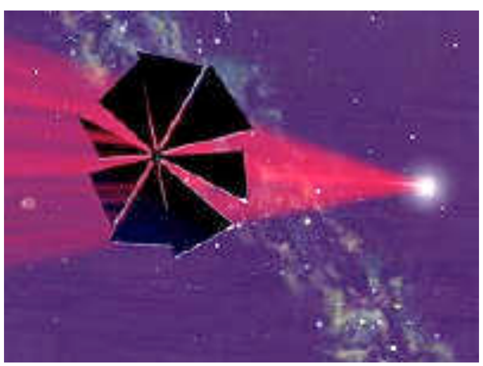


Not radiation pressure!!



Comet tails

Solar sails?



From the Planetary Society

Sun radiation: I= 1 kW/m^2

Area 1km^2 => F=IA/c=3.3 mN

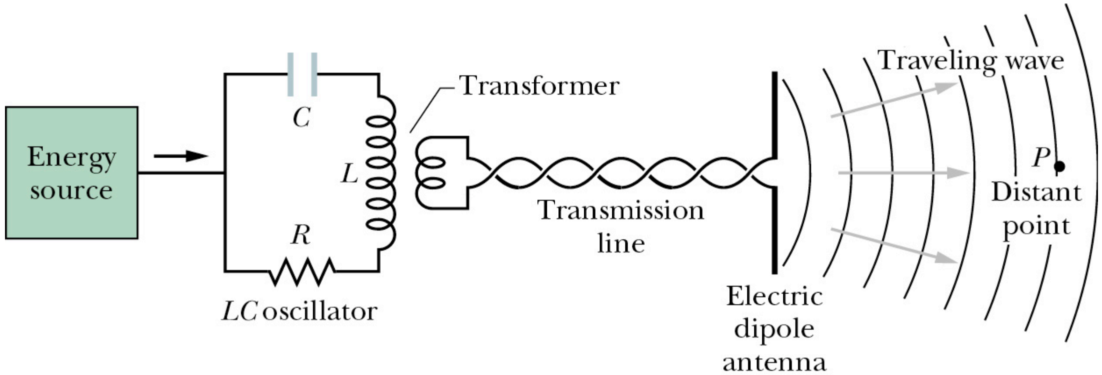
Mass m=10 kg => a=F/m=3.3 x 10^{-4} m/s^2

When does it reach 10mph=4.4 m/s?

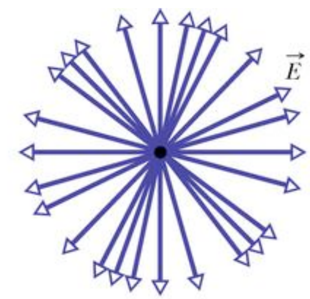
V=at => t=V/a=1.3 x 10^4 s=3.7 hrs

EM waves: polarization

Radio transmitter:



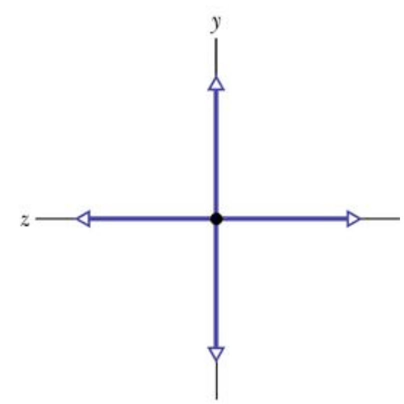
If the dipole antenna is vertical, so will be the electric fields. The magnetic field will be horizontal



(a)

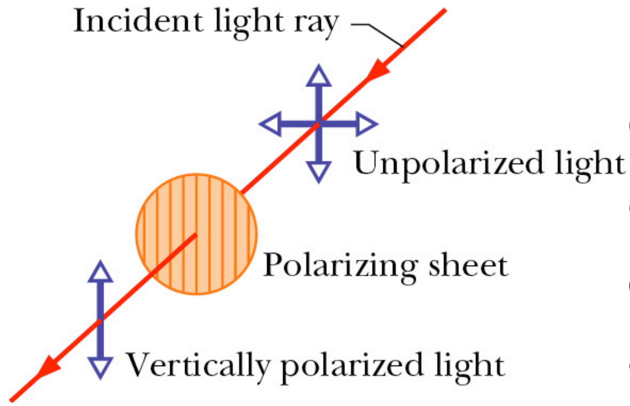
The radio wave generated is said to be "polarized"

In general light sources produce "unpolarized waves" emitted by atomic motions in random directions



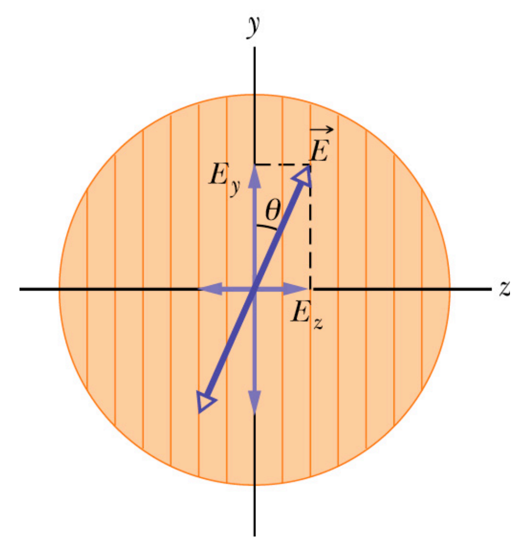
(b)

EM waves: polarization



Completely unpolarized light will have equal components in horizontal and vertical directions. Therefore running the light through a polarizer will cut the intensity in half: $I=I_0/2$

When polarized light hits a polarizing sheet, only the component of the field aligned with the sheet will get through.

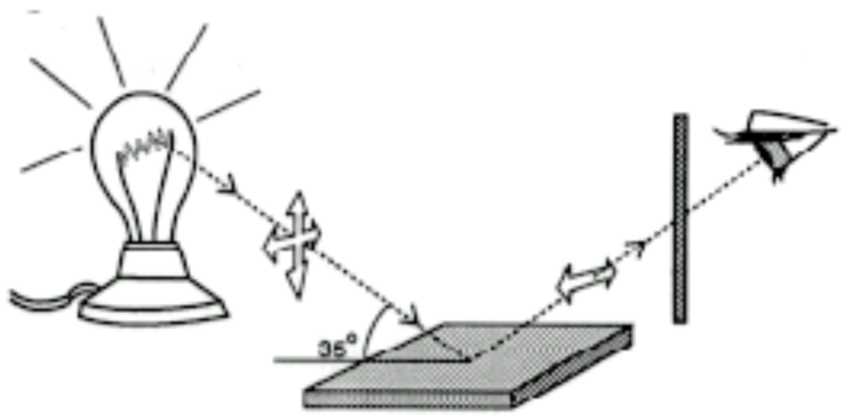


$$E_y = E\cos(\theta)$$

And therefore: $$I = I_0 \cos^2\theta$$

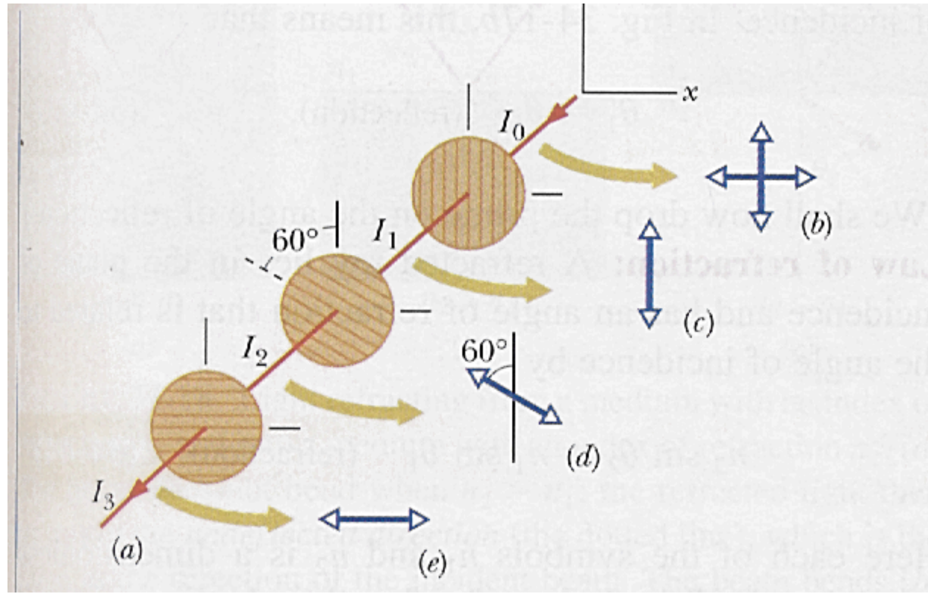
Example

- Polarized sunglasses cut the intensity in half: $I=I_0/2$
- They cut the horizontally polarized light from glare (reflections on roads, cars, etc)



Example

Initially unpolarized light of intensity I_0 is sent into a system of three polarizers as shown. What fraction of the initial intensity emerges from the system? What is the polarization of the exiting light?



- Through the first polarizer: unpolarized to polarized, so $I_1=\frac{1}{2}I_0$.
- Into the second polarizer, the light is now vertically polarized. Then, $I_2=I_1\cos^2 60^o = 1/4\ I_1 = 1/8\ I_0$.
- Now the light is again polarized, but at 60^o. The last polarizer is horizontal, so $I_3=I_2\cos^2 30^o = 3/4\ I_2 = 3/32\ I_0 = 0.094\ I_0$.
- The exiting light is horizontally polarized, and has 9% of the original amplitude.

Summary

- Variation of power of **spherical waves**

$$I = \frac{\text{power}}{\text{area}} = \frac{P_S}{4\pi r^2}$$

- **Radiation pressure**:

$$p_r = \frac{I}{c} \text{ (total absorption)}, \quad p_r = \frac{2I}{c} \text{ (total reflection)}$$

- **Unpolarized** light through polarizer: $I = I_0/2$
- When polarized light hits a polarizer:

$$I = I_0 \cos^2 \theta$$