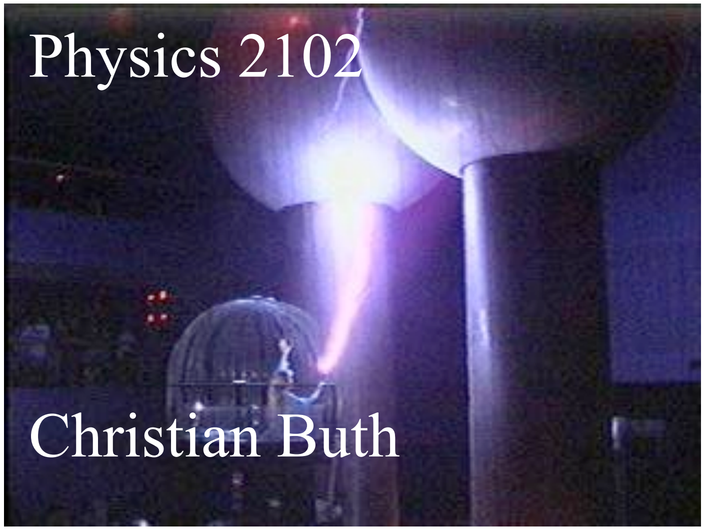
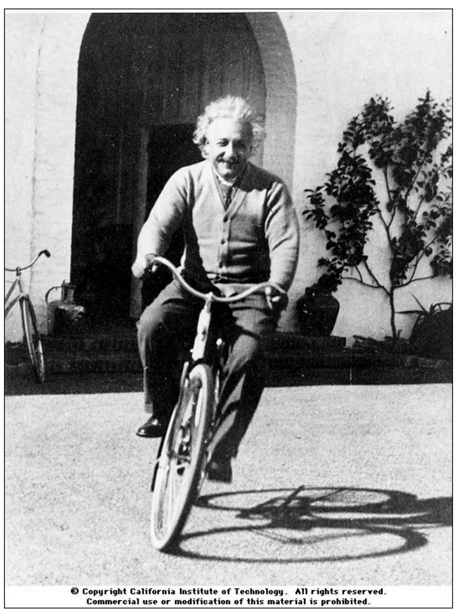


James Clerk Maxwell (1831-1879)

Lecture 32

Review Lectures 21-31



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Lecture 21

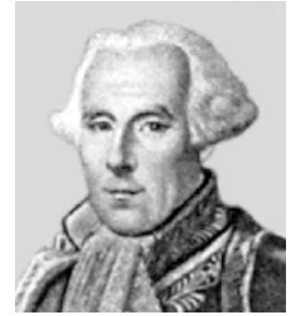
- **Cyclotrons** and **synchrotrons** to accelerate particles
- Wires carrying currents produce forces on each other
- Parallel currents **attract**, antiparallel currents **repel**
- For a straight wire: $\vec{F}_B = i\vec{L} \times \vec{B}$, generally: $\vec{F}_B = i\int d\vec{L} \times \vec{B}$
- **Current loop** is **magnetic dipole;** in uniform magnetic field

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad \vec{\mu} = (NiA)\hat{n} \qquad U = -\vec{\mu} \cdot \vec{B}$$

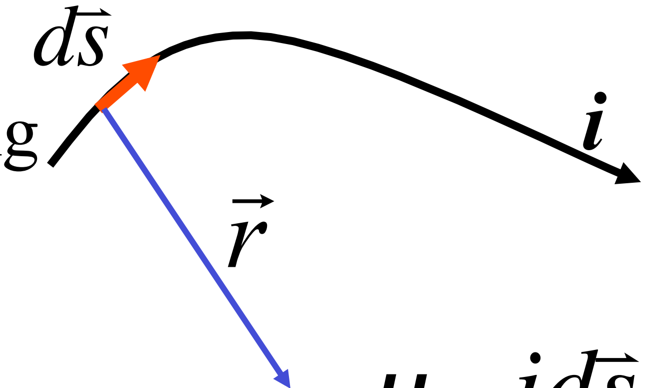
- **Right-hand rule** gives direction of moment
- **Magnetic potential energy** of magnetic dipole in uniform magnetic field

Jean-Baptiste
Biot (1774-1862)

Biot-Savart Law



Felix Savart
(1791-1841)



- Quantitative law for computing the magnetic field from any electric current
- Choose a differential element of wire of length ***ds*** and carrying a current ***i***
- The field ***dB*** from this element at a point located by the vector ***r*** is given by the Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^3}$$

$\mu_0 = 4\pi x 10^{-7}$ Tm/A
(permeability constant)

Compare with $d\vec{E} = \frac{dq}{4\pi\varepsilon_0} \frac{\vec{r}}{r^3}$

Lecture 22

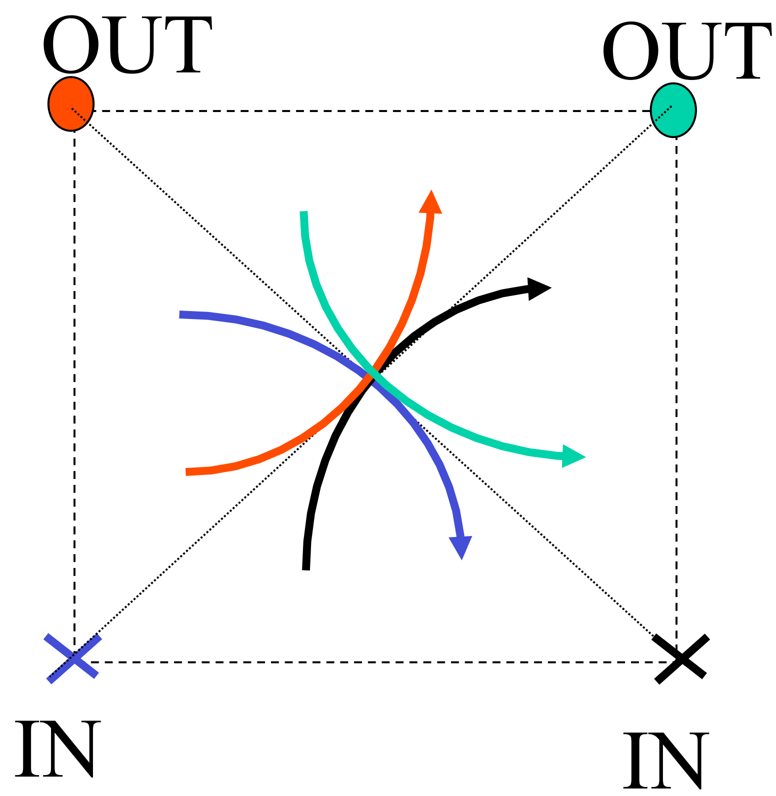
- Magnetic fields from currents from **Biot-Savart's law**:

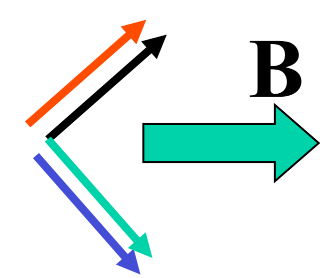
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3}$$

- Two **right-hand rules** for direction of magnetic field
- **Straight currents** produce circular magnetic fields:
  $B = \mu_0 i / 2\pi r$
- Current through **circular arc** produce magnetic field at center:
  $B = \mu_0 i \Phi / 4\pi r$

Superposition

- Magnetic fields (like electric fields) can be "superimposed" -- just do a vector sum of *B* from different sources
- The figure shows four wires located at the 4 corners of a square. They carry equal currents in directions indicated
- What is the direction of B at the center of the square?





Lecture 23

- Wires carrying currents produce forces on each other: **parallel currents attract**, antiparallel currents repel
- Force between wires

$$F_{21} = L I_2 B_1 = \frac{\mu_0 L I_1 I_2}{2\pi a}$$

- **Ampere's law** analog to Gauss' law for electric fields:

The line integral $\oint \vec{B} \cdot d\vec{s}$ of the magnetic field $\vec{B}$ along any closed path is equal to the total current enclosed inside the path multiplied by μ_0.

Lecture 24

- Inside a wire with uniform current density:

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

- **Solenoid** is tightly wound helical wire $B = \mu_0 n i$
- **Toroid** is a **doughnut** shaped coil

$$B = \frac{\mu_o N i}{2\pi r}$$

- Magnetic field of circular loop is a magnetic dipole

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{(R^2 + z^2)^{3/2}} \approx \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$$

- **Magnetic dipole moment** of magnitude $\mu = NiA$

Lecture 25

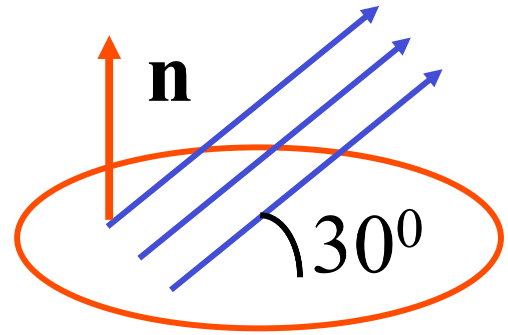
- Magnetic flux: $\Phi_B = \int \vec{B} \cdot d\vec{A}$
- Faradays' law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$

An emf is induced in a loop when the number of magnetic field lines that pass through the loop is changing.

- Negative sign in Faradays' law from **Lenz rule**:

An induced current has a direction such that the magnetic field due to the induced current opposes the change in the magnetic flux that induces the current.

Example



- A closed loop of wire encloses an area of 1 m^2 in which a <u>uniform</u> magnetic field exists at 30^0 to the PLANE of the loop. The magnetic field is DECREASING at a rate of 1T/s. The resistance of the wire is 10 Ω.
- What is the induced current?

$$\Phi_B = \int_S \vec{B} \cdot \hat{n} dA$$

$$= BA\cos(60^0) = \frac{BA}{2}$$

$$|EMF| = \frac{d\Phi_B}{dt} = \frac{A}{2}\frac{dB}{dt}$$

$$i = \frac{EMF}{R} = \frac{A}{2R}\frac{dB}{dt}$$

$$i = \frac{(1m^2)}{2(10\Omega)}(1T/s) = 0.05A$$

Is it
…clockwise or
…counterclockwise?

Lecture 26

- Lenz's rule different formulation of **energy conservation**
- Thermal energy and mechanical energy are **equal** when pulling a conducting loop through a magnetic field
- A pendulum in magnetic fields is slowed down due to induced **eddy currents**

Lecture 27

- Alternative version of **Faradays' law**:

A changing magnetic field produces an electric field.

$$\oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

- The **electric field** inside / outside a solenoid with increasing current

$$E = \frac{r}{2}\frac{dB}{dt}$$

$$E = \frac{R^2}{2r}\frac{dB}{dt}$$

Lecture 28

- Notion of electric potential **does not work** for electric fields produced by induction
- **Inductance** of a solenoid

$$\Phi_B = NAB = Li$$

- SI unit **henry**
- **Self induction**: an EMF appears in any coil in which the current is changing: $$EMF = -L\frac{di}{dt}$$
- **Direction** of self-induced EMF from Lenz's law
- "Charging" an inductor: $i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$
- "Discharging" an inductor: $i = \frac{\mathcal{E}}{R}e^{-\frac{Rt}{L}}$

Lecture 29 / 1

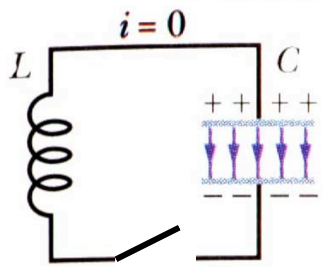
- **Energy** stored in a magnetic field

$$U_B = \frac{Li^2}{2}$$

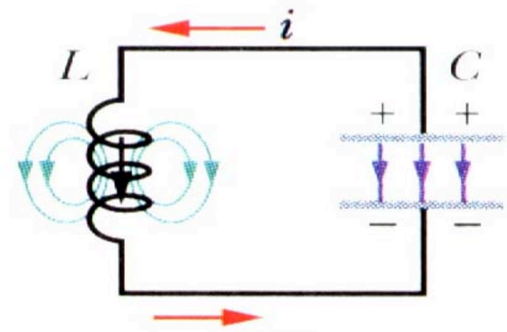
- **Energy density** in magnetic field

$$u_B = \frac{B^2}{2\mu_0}$$

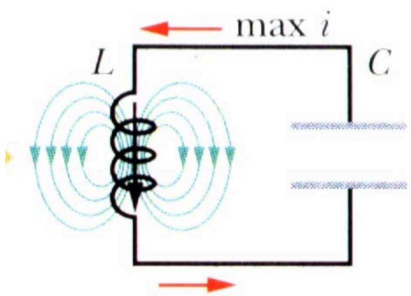
An electromagnetic oscillator



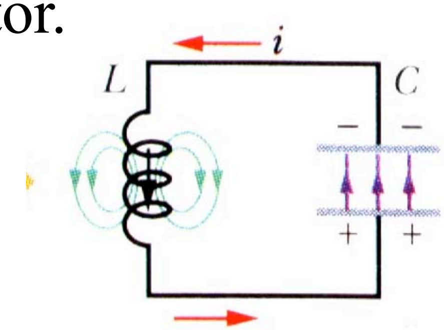
Capacitor initially charged. Initially, current is zero, energy is all stored in the capacitor.



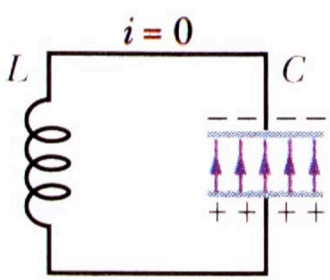
A current gets going, energy gets split between the capacitor and the inductor.



Capacitor discharges completely, yet current keeps going. Energy is all in the inductor.



The magnetic field on the coil starts to collapse, which will start to recharge the capacitor.



Finally, we reach the same state we started with (with opposite polarity) and the cycle restarts.

Lecture 29 / 2

- Combining a capacitor and an inductor produces an **electrical oscillator**
- Total energy in circuit is **conserved**: switches between capacitor (electric field) and inductor (magnetic field)
- Differential equation of **LC circuit**:

$$0 = L\frac{d^2q}{dt^2} + \frac{q}{C}$$

$$q = q_0 \cos(\omega t + \phi_0)$$

- **Natural frequency** of oscillator is $\omega = 1/\sqrt{LC}$

Lecture 30

- Resistor in RLC circuit dissipates energy

$$q(t) = Qe^{-Rt/2L}\cos\left(\omega' t + \phi\right)$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

- **Alternating current** (ac)

$$\mathcal{E} = \mathcal{E}_m \sin(\omega_d t) \qquad i(t) = I\sin(\omega_d t)$$

- Specified as **root-mean-square** (rms)

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} \qquad V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

Lecture 31

- Transformer equation

$$V_S = \frac{N_S}{N_P} V_P \qquad\qquad i_S = \frac{N_P}{N_S} i_P$$

- **Maxwell's equations** are:
  - Gauss law for electric fields
  - Gauss law for **magnetic** fields $\oint B \bullet dA = 0$
  - **Ampere-Maxwell** law

$$\oint_C \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

  - Faraday's law