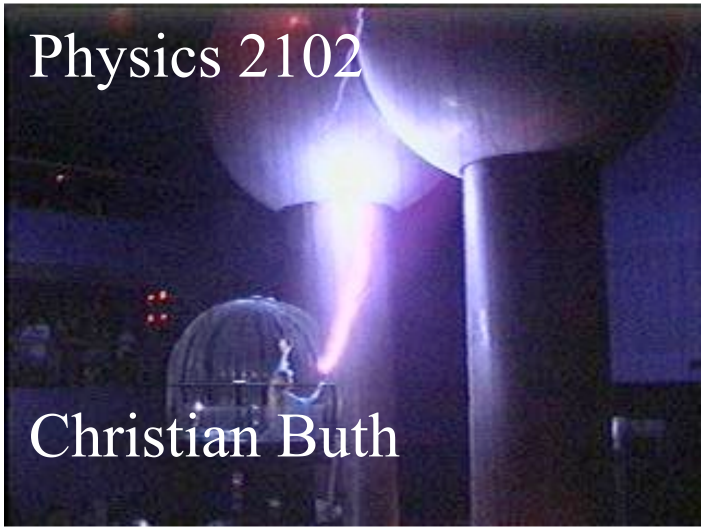
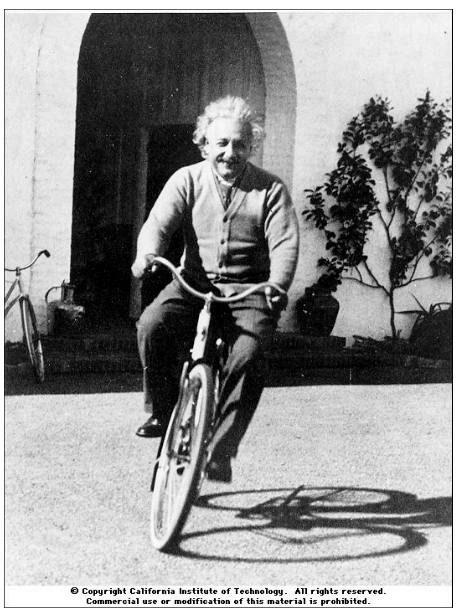


James Clerk Maxwell (1831-1879)

Lecture 31

Maxwell's equations

the dawn of the 20th century
revolution in physics



03/30/2009

Review

- Resistor in RLC circuit dissipates energy

$$q(t) = Qe^{-Rt/2L}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

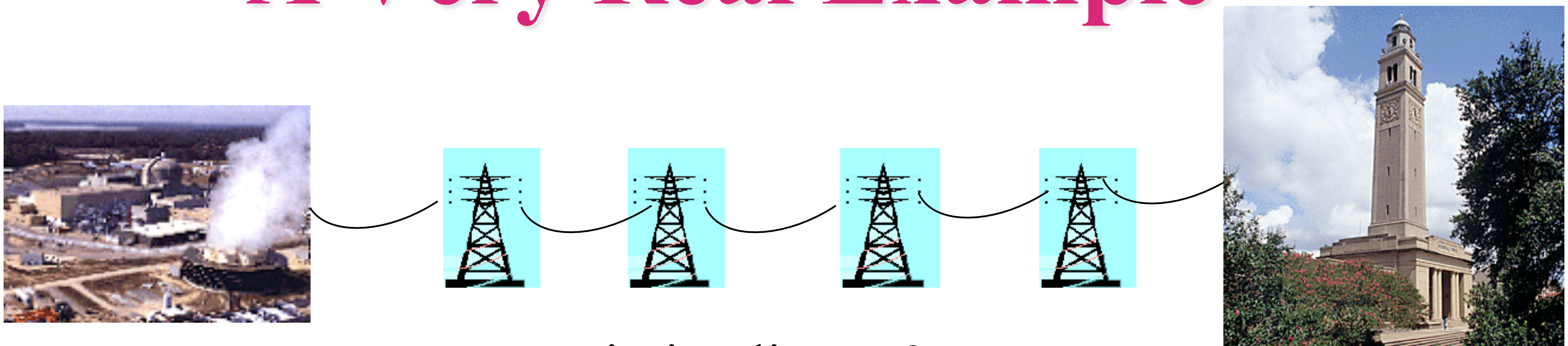
- **Alternating current** (ac)

$$\mathcal{E} = \mathcal{E}_m \sin(\omega_d t) \qquad i(t) = I\sin(\omega_d t)$$

- Specified as **root-mean-square** (rms)

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} \qquad V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

A Very Real Example



River Bend power plant

Transmission lines 735KV
Distance ~30 miles ~50km
Resistance 0.22Ω/km
Power capacity 936 MW

LSU

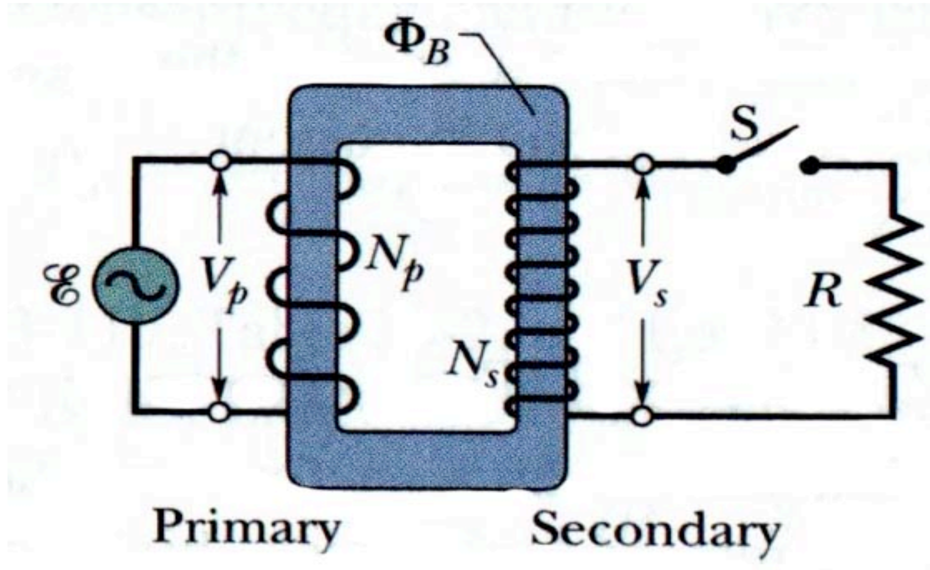
Current: P=IV, I=P/V=936MW/735kV=1300 A (!)
Power dissipated in wires:

$$P_{lost}=I^2R=(1300A)^2 x 0.22\Omega/km \times 50km=19\ MW\ (\sim 2\%\ of\ 936)$$

If the power delivered is constant, we want the highest voltage and the lowest current to make the delivery efficient!
At home, however, we don’t want high voltages! We use transformers.

Transformers

Two coils ("primary and secondary") sharing the same magnetic flux.



Faraday's law:

$$\frac{d\Phi}{dt} = \text{emf per turn} \qquad = \frac{V_P}{N_P} = \frac{V_S}{N_S} \qquad \boxed{V_S = \frac{N_S}{N_P} V_P}$$

You can get any voltage you wish just playing with the number of turns. For instance, the coil in the ignition system of a car goes from 12V to thousands of volts. Or the transformers in most consumer electronics go from 110V to 6 or 12 V.

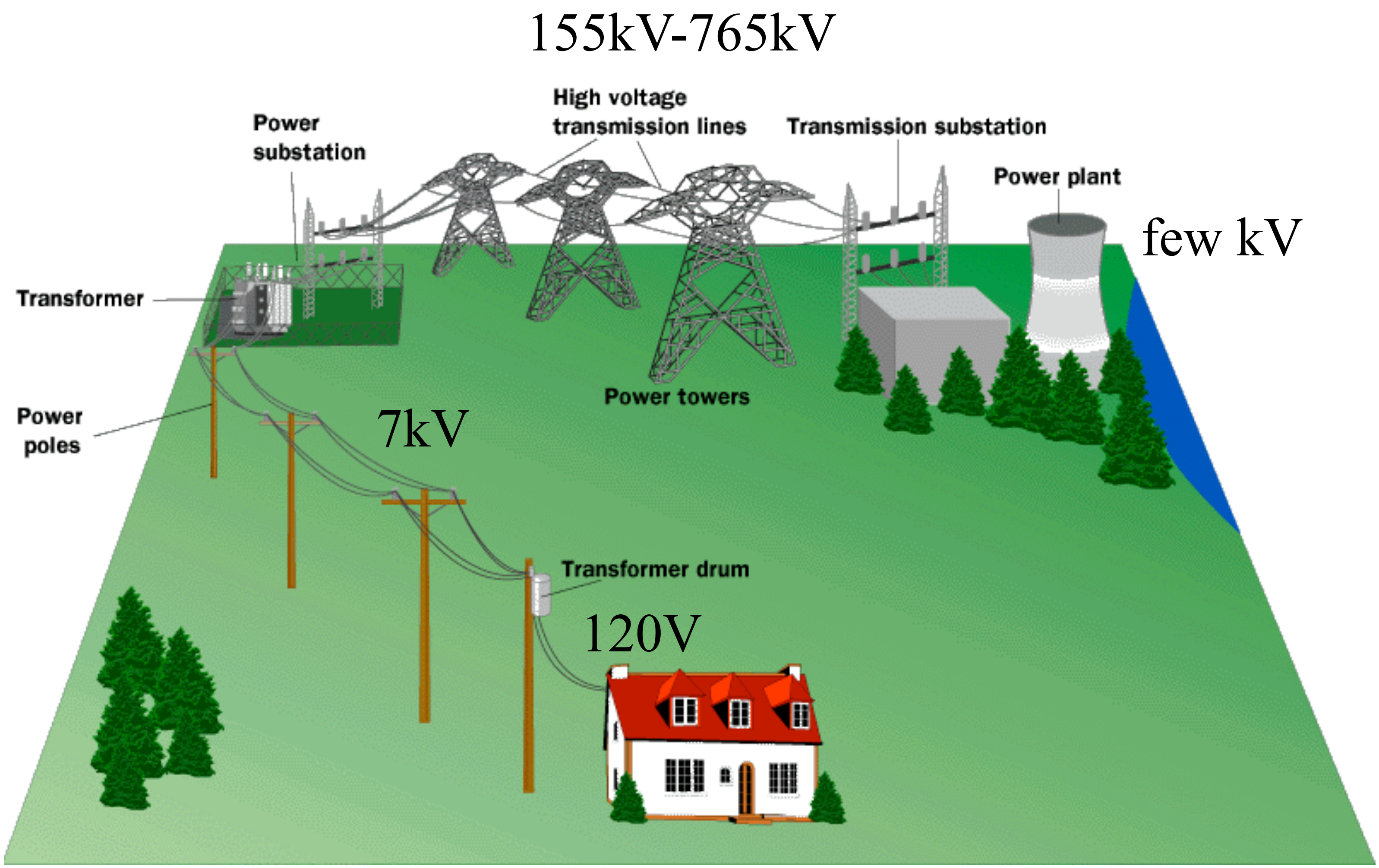
$$\text{Energy is conserved}: \; i_S = \frac{N_P}{N_S} i_P$$

$$I_P V_P = I_S V_S$$

What you gain (lose) in voltage you lose (gain) in current

From the Power Plant to Your Home

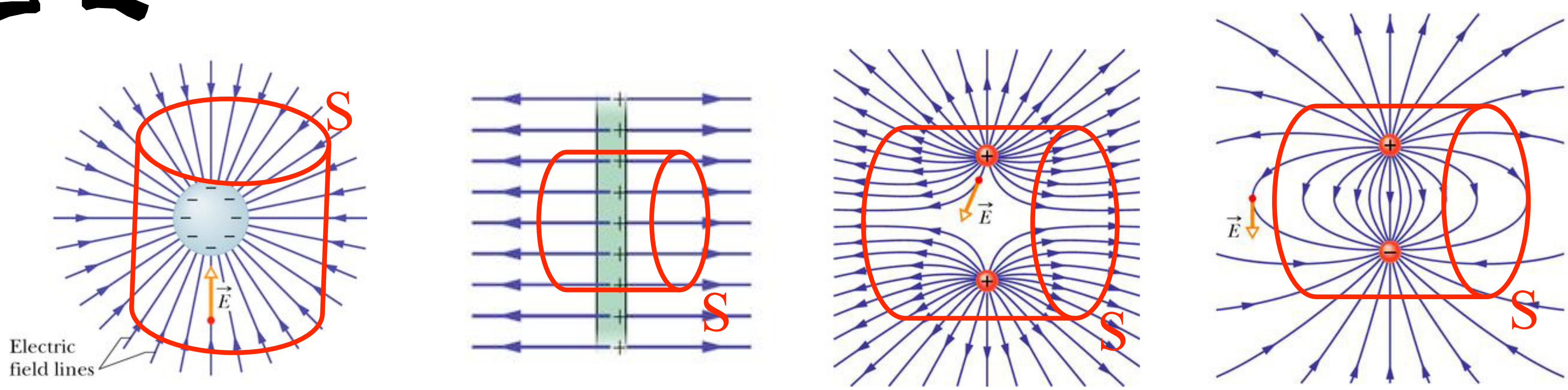
http://www.howstuffworks.com/power.htm: The Distribution Grid



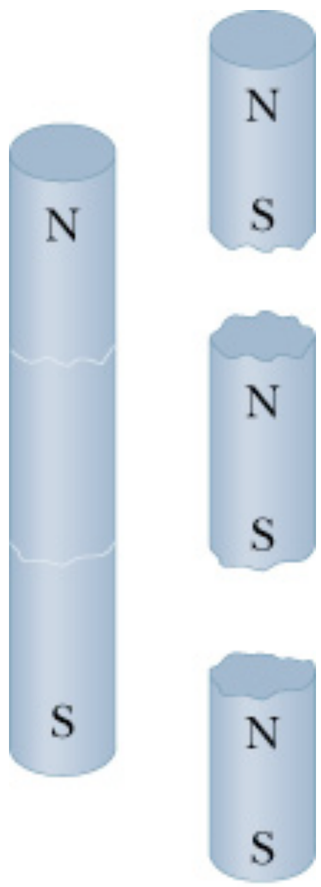
Gauss' Law for the Electric Field

Charges produce electric fields,
field lines start and end in charges

$$\oint_S E \bullet dA = q/\varepsilon_0$$



Gauss' Law for the Magnetic Field



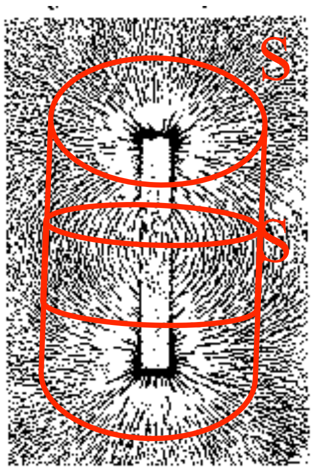
- Magnetic poles **cannot** be separated
- Cut magnet into pieces yields **two** magnets with North and South pole

The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist as far as we know.

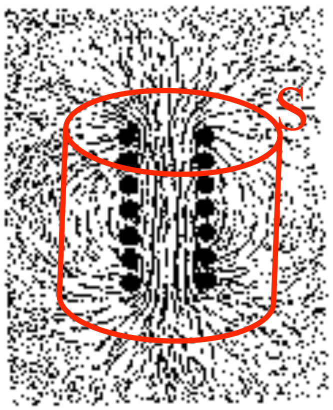
Gauss' law for magnetism

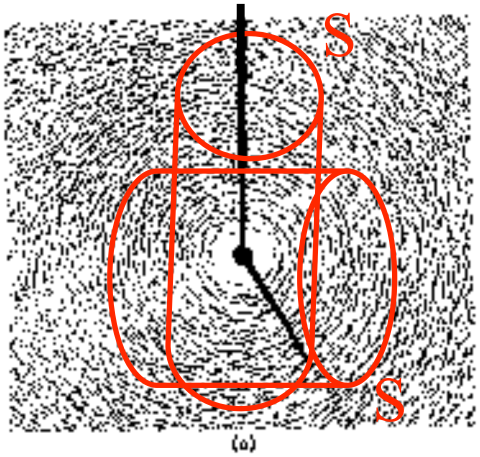
Field lines are closed

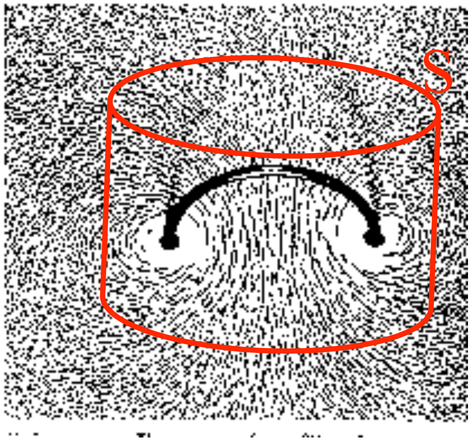
or, there are no magnetic monopoles



$$\oint_S B \bullet dA = 0$$



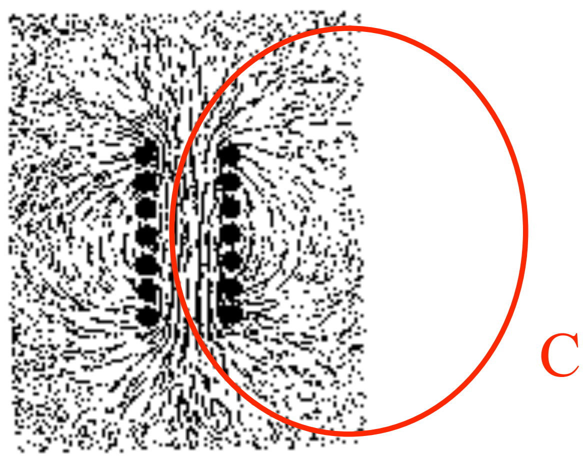




Ampere's law

Electric currents produce magnetic fields

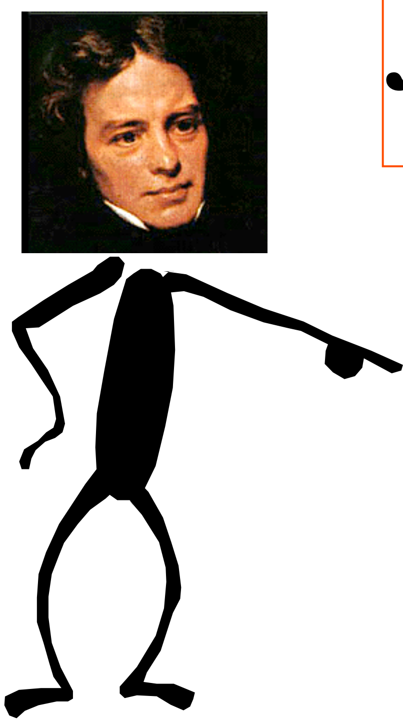
$$\oint_C B \bullet ds = \mu_0 i$$

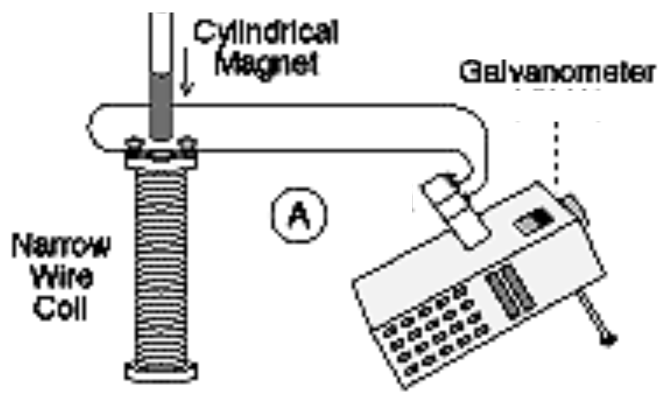


Faraday's law

Changing magnetic fields produce ("induce")
electric fields

$$\oint_C E \bullet ds = -\frac{d}{dt}\int_S B \bullet dA$$





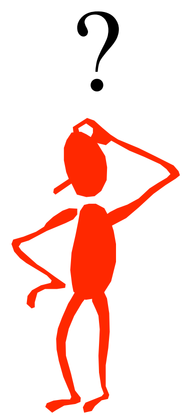
No charges or currents

$$\oint_S E \bullet dA = 0$$

q=0

$$\oint_S B \bullet dA = 0$$

i=0

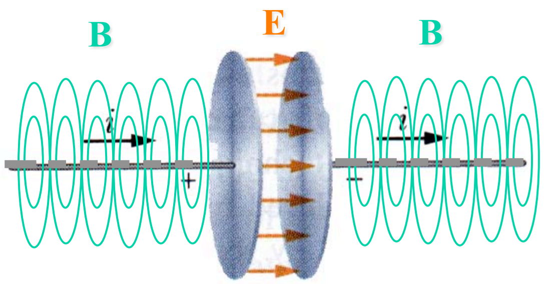


$$\oint_C B \bullet ds = 0$$

…very suspicious…!

$$\oint_C E \bullet ds = -\frac{d}{dt}\int_S B \bullet dA$$

Something is not right…



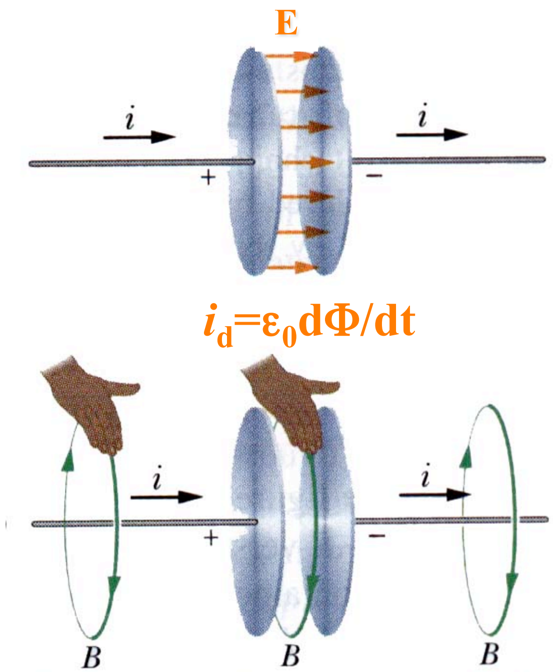
If we are charging a capacitor, there is a current left and right of the capacitor.

Thus, there is the same magnetic field right and left of the capacitor, with circular lines around the wires.

But no magnetic field inside the capacitor?

With a compass, we can verify there is indeed a magnetic field, equal to the field elsewhere.

But there is no current producing it! ?



Maybe we can make it right…

We calculate the magnetic field produced by the currents at left and at right using Ampere's law :

$$\oint_C B \bullet ds = \mu_0 i$$

We can write the current as:

$$i = \frac{dq}{dt}$$

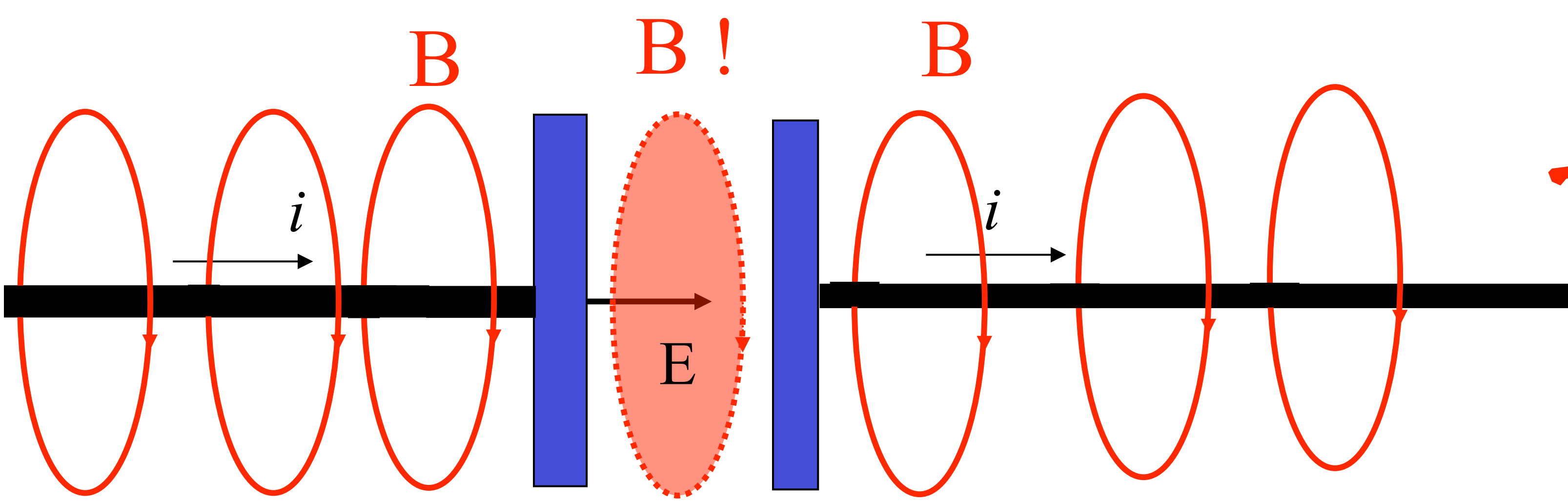
$q = CV$ $\quad C = \varepsilon_0 A/d$ $\quad V = Ed$ $\quad \Phi_E = \int E \bullet dA = EA$

Displacement current

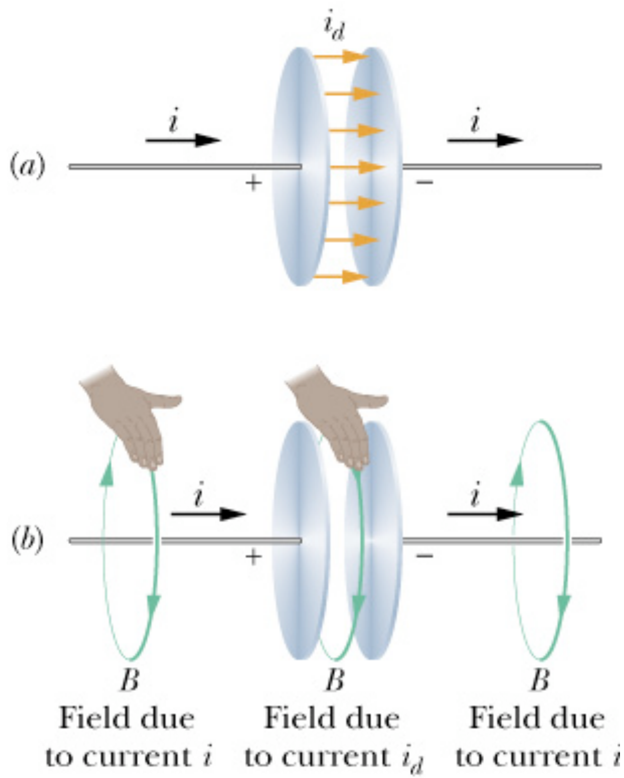
$$\oint_C B \bullet ds \neq 0$$

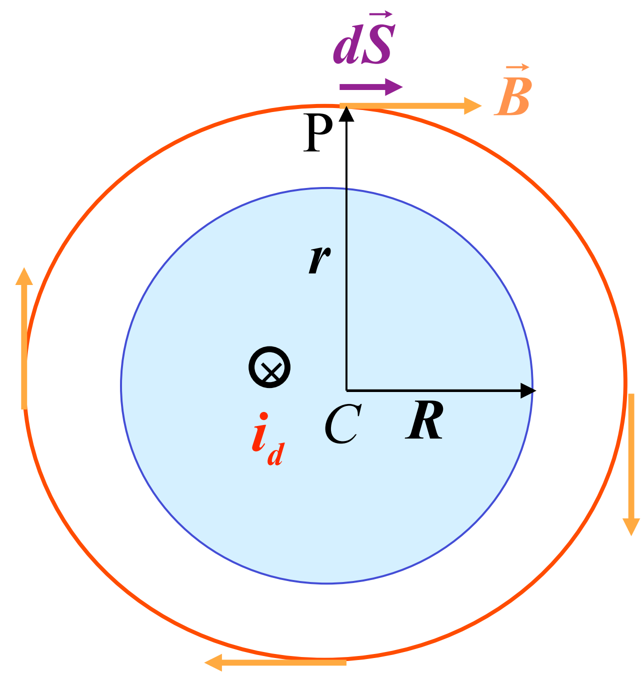
Maxwell proposed it, and it was confirmed.

$$\oint_C B \bullet ds = \mu_0 \varepsilon_0 \frac{d}{dt} \int_S E \bullet dA = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 i_d$$



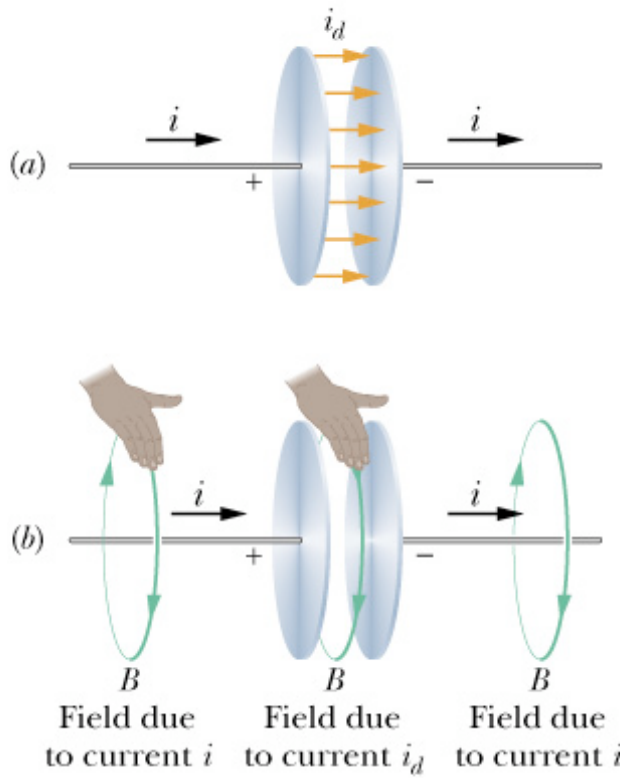
Magnetic Field Outside Plates





- Magnetic field **tangent** to loop and **constant** magnitude $B = \dfrac{\mu_0 i_d}{2\pi r}$
- Ampere-Maxwell's law:

$$\oint_C \vec{B} \bullet d\vec{s} = \oint_C B\, ds \cos 0 = B \oint_C ds = 2\pi r B = \mu_0 i_{\text{d, enc}} = \mu_0 i_{\text{d}}$$



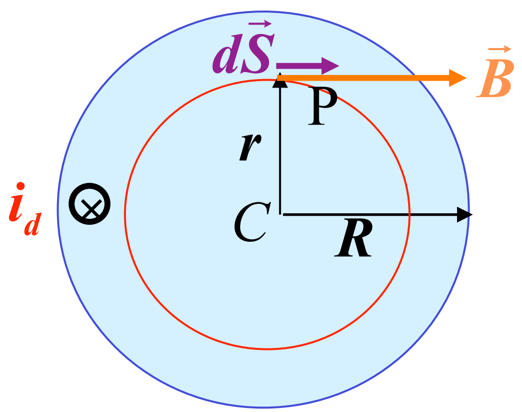
Magnetic Field Inside Plates

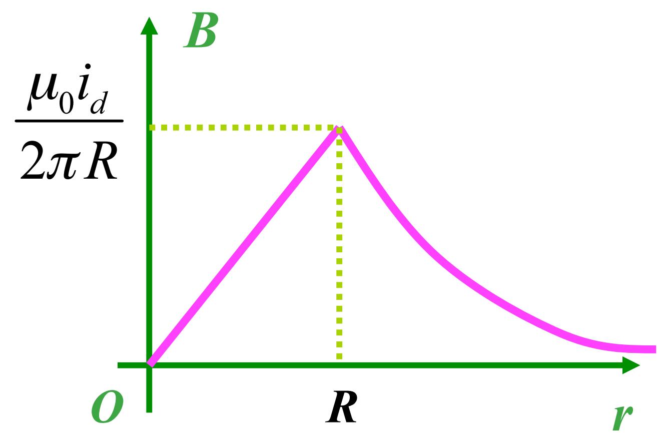
- Ampere-Maxwell law:

$$\oint_C \vec{B} \bullet d\vec{s} = 2\pi r B = \mu_0 i_{d,\ enc}$$

- **Enclosed** displacement current:

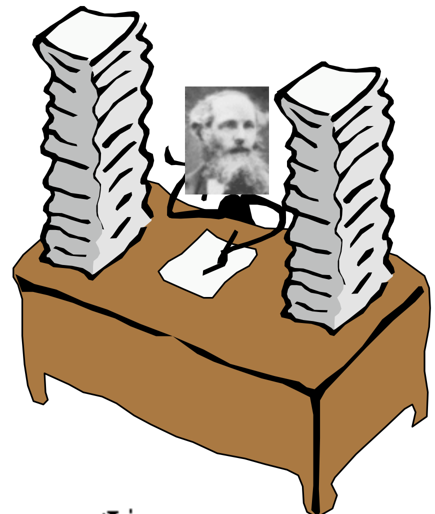
$$i_{d,enc} = i_d \frac{\pi r^2}{\pi R^2} \qquad B = \frac{\mu_0 i_d}{2\pi R^2} r$$





"Maxwell" equations

$$\oint_S E \bullet dA = q/\varepsilon_0$$



$$\oint_S B \bullet dA = 0$$

$$\oint_C B \bullet ds = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i$$

$$\oint_C E \bullet ds = -\frac{d}{dt}\int_S B \bullet dA$$

Summary

- Transformer equation

$$V_S = \frac{N_S}{N_P} V_P \qquad i_S = \frac{N_P}{N_S} i_P$$

- **Maxwell's equations** are:
  - Gauss law for electric fields
  - Gauss law for **magnetic** fields $\oint B \bullet dA = 0$
  - **Ampere-Maxwell** law

$$\oint_C \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

  - Faraday's law