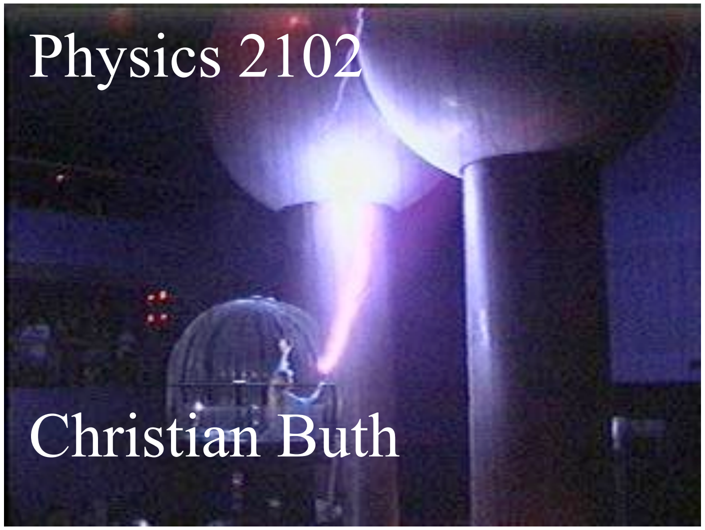
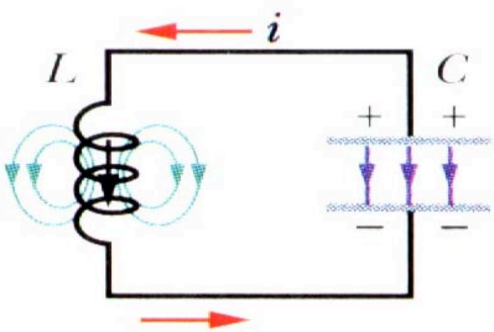
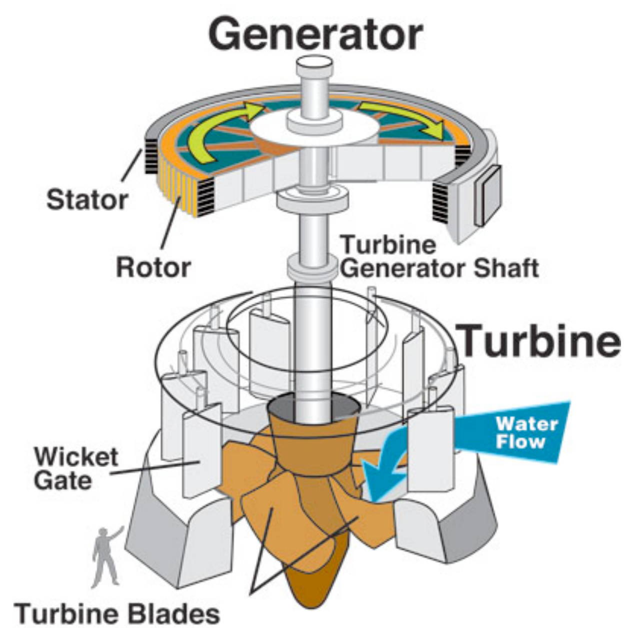


Lecture 30

Alternating Current

03/27/2009





Review 1

- **Energy** stored in a magnetic field

$$U_B = \frac{Li^2}{2}$$

- **Energy density** in magnetic field

$$u_B = \frac{B^2}{2\mu_0}$$

Review 2

- Combining a capacitor and an inductor produces an **electrical oscillator**
- Total energy in circuit is **conserved**: switches between capacitor (electric field) and inductor (magnetic field)
- Differential equation of **LC circuit**:

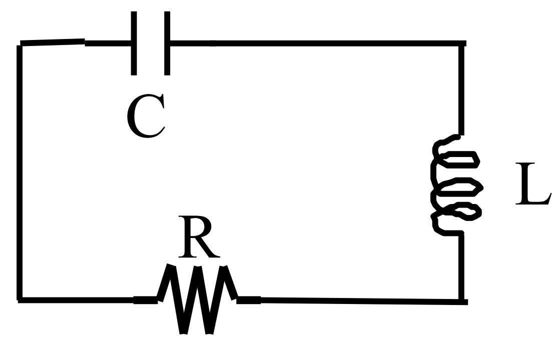
$$0 = L\frac{d^2q}{dt^2} + \frac{q}{C}$$

$$q = q_0 \cos(\omega t + \phi_0)$$

- **Natural frequency** of oscillator is $\omega = 1/\sqrt{LC}$

Damped LC Oscillator

- **Ideal LC circuit** without resistance:



$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

- Oscillations go on **for ever**: **$\omega = (LC)^{-1/2}$**

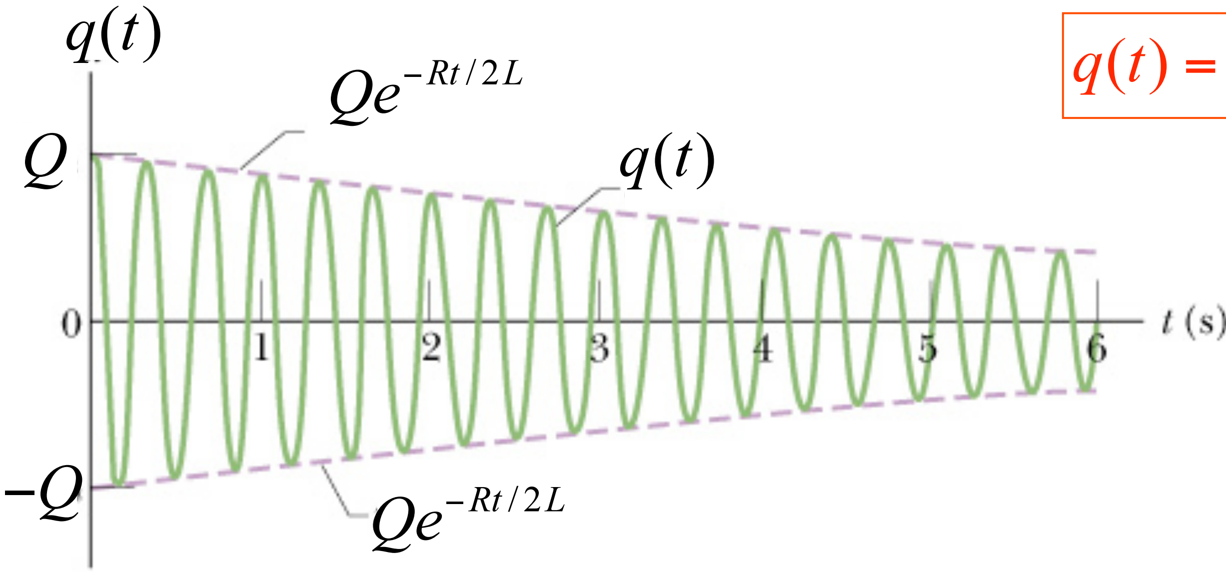
- **Real RLC circuit** has resistance, **dissipates energy**: oscillations die out, or are "damped"

$$U = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C}$$

$$\frac{dU}{dt} = \frac{1}{2}L\left(2i\frac{di}{dt}\right) + \frac{1}{2C}\left(2q\frac{dq}{dt}\right) = -i^2R$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

Damped Oscillations



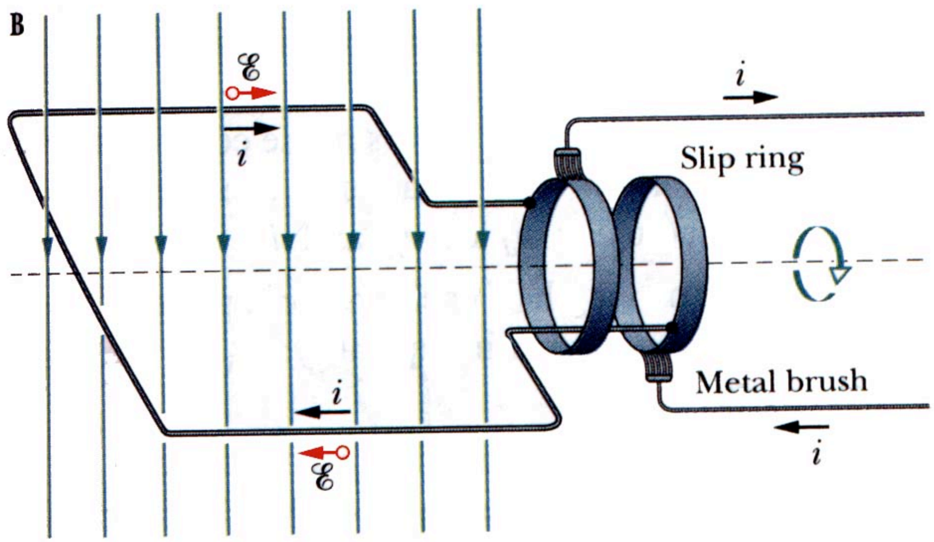
$$q(t) = Qe^{-Rt/2L}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$U_{\max} = \frac{Q^2}{2C} e^{-\frac{Rt}{L}}$$

- Math is complicated!
- Frequency of oscillator **shifts away** from $\omega = (LC)^{-1/2}$
- **Peak charge** decays with time constant = 2L/R
- Small damping: **peak energy** decays with time constant L/R

Alternating Current



We have studied that a loop of wire, spinning in a constant magnetic field will have an induced emf that oscillates with time,

$$\mathcal{E} = \mathcal{E}_m \sin(\omega_d t)$$

That is, it is an AC generator

AC's are very easy to generate, they are also easy to amplify and decrease in voltage. This in turn makes them easy to send in distribution grids like the ones that power our homes

- The **driving angular frequency** is ω_d
- **Alternating current** (ac) is: $i(t) = I \sin(\omega_d t)$

Forced Oscillations

To keep oscillations going we need to drive the *RLC* circuit with an external emf that produces a current that goes back and forth

Notice that there are two frequencies involved: one at which the circuit would oscillate "naturally". The other is the frequency at which we **drive** the oscillation

However, the "natural" oscillation usually dies off quickly (exponentially) with time. Therefore in the long run, circuits actually oscillate with the frequency at which they are driven. (All this is true for the gentleman trying to make the lady swing back and forth in the picture too).

Power in AC Circuits

$V = V_m \sin(\omega_d t)$

$I = I_m \sin(\omega_d t)$

Power **dissipated** by R:

$P = I^2 R$

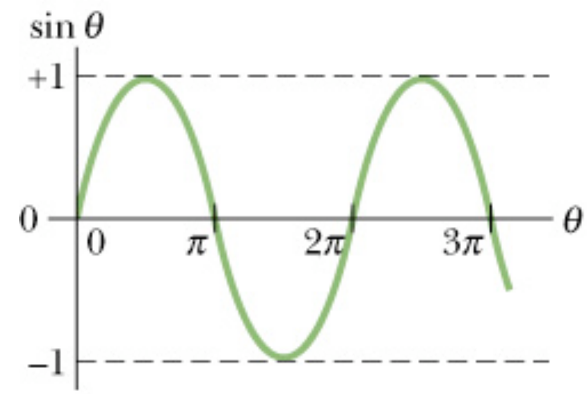
$P = I_m^2 R \sin^2(\omega_d t)$

$P_{average} = (1/2)\, I_m^2 R$

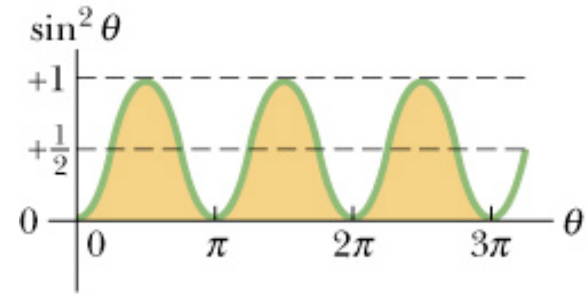
$= I_{rms}^2 R$

Root-means-square (rms):

$$I_{rms} = \frac{I_m}{\sqrt{2}} \qquad V_{rms} = \frac{V_m}{\sqrt{2}}$$



(a)



(b)

Summary

- Resistor in RLC circuit dissipates energy

$$q(t) = Qe^{-Rt/2L}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

- **Alternating current** (ac)

$$\mathcal{E} = \mathcal{E}_m \sin(\omega_d t) \qquad i(t) = I\sin(\omega_d t)$$

- Specified as **root-mean-square** (rms)

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} \qquad V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$