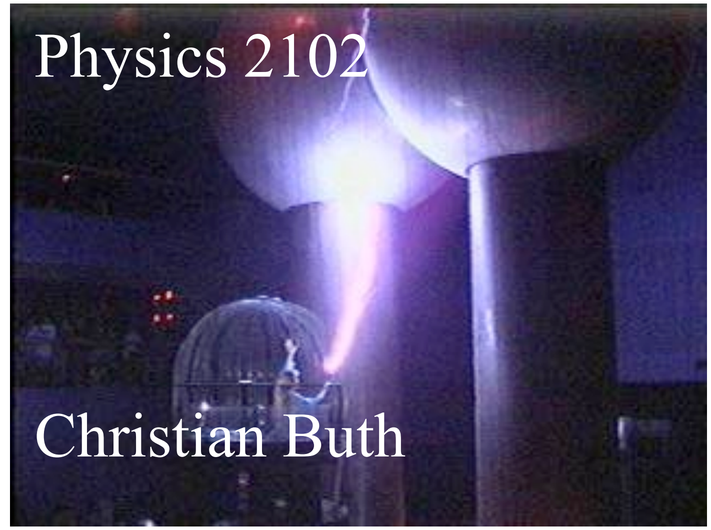


Physics 2102
Lecture 28

Inductors and RL Circuits

03/23/2009

Nikola Tesla

Review

- Alternative version of **Faradays' law**:

A changing magnetic field produces an electric field.

$$\oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

- The **electric field** inside / outside a solenoid with increasing current

$$E = \frac{r}{2}\frac{dB}{dt} \qquad E = \frac{R^2}{2r}\frac{dB}{dt}$$

Validity of Electric Potential

- **Condition** for a potential:

$$\oint_C \vec{E} \cdot d\vec{s} = 0$$

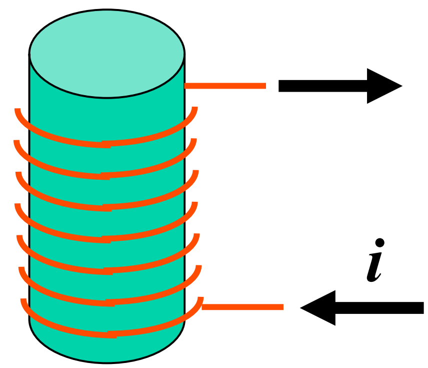
- Electric potential has meaning **only** for electric fields produced by **static charges**; only then line integral is independent of path and it holds:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

- Potential has **no meaning** for electric fields from induction

“Self”-Inductance of a solenoid

- Solenoid of cross-sectional area ***A***, length ***l***, total number of turns ***N***, turns per unit length ***n***
- Field inside solenoid = $\mu_0\, n\, i$
- Field outside ~ 0

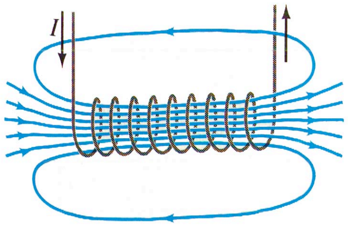


$$\Phi_B = NAB = NA\mu_0 ni = Li$$

L = “inductance” $= \mu_0 NAn = \mu_0 \frac{N^2}{l} A$

Inductors: Solenoids

Inductors are with respect to the magnetic field what capacitors are with respect to the electric field. They "pack a lot of field in a small region". Also, the higher the current, the higher the magnetic field they produce



Capacitance → how much **potential** for a given charge: **Q=CV**

Inductance → how much **magnetic flux** for a given current: **Φ=L*i***

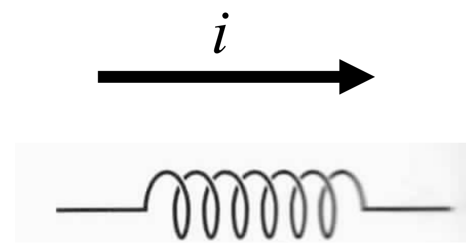
Using Faraday's law: $$EMF = -L\frac{di}{dt}$$

Units: $$[L] = \frac{\text{Tesla}\cdot\text{m}^2}{\text{Ampere}} \equiv \text{H} \ \ \text{(Henry)}$$

Joseph Henry
(1799-1878)

$$EMF = -L\frac{di}{dt}$$

Example



- The current in a 10 H inductor is decreasing at a steady rate of 5 A/s.
- If the current is as shown at some instant in time, what is the magnitude and direction of the induced EMF?

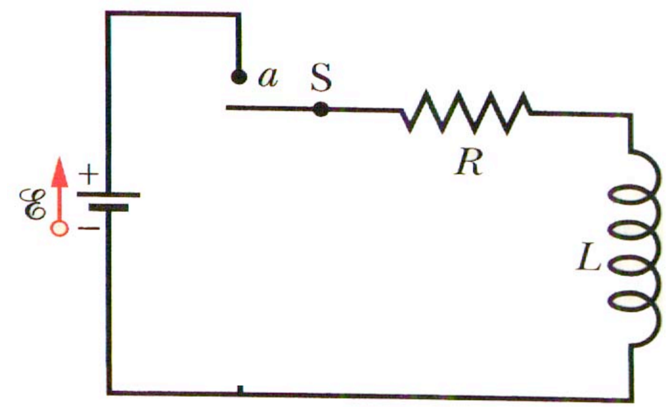
(a) 50 V →

(b) 50 V ←

- Magnitude = (10 H)(5 A/s) = 50 V
- Current is decreasing
- Induced emf must be in a direction that OPPOSES this change.
- So, induced emf must be in same direction as current

The RL circuit

- Set up a single loop series circuit with a battery, a resistor, a solenoid and a switch.
- Describe what happens when the switch is closed.
- Key processes to understand:
  - What happens JUST AFTER the switch is closed?
  - What happens a LONG TIME after switch has been closed?
  - What happens in between?



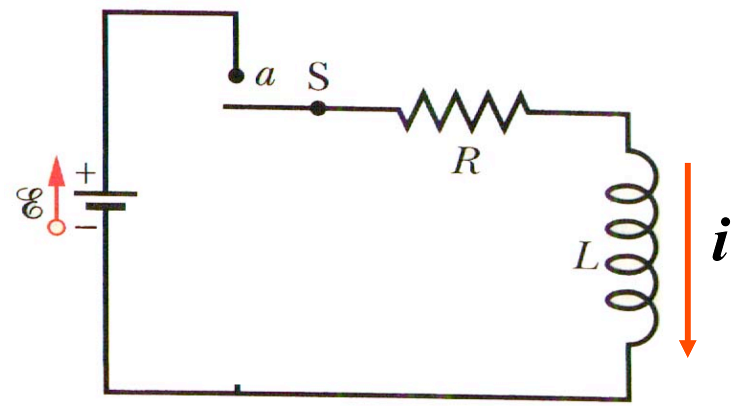
<u>Key insights:</u>

- If a circuit is not broken, one cannot change the CURRENT in an inductor instantaneously!
- If you wait long enough, the current in an RL circuit stops changing!

At t=0, a capacitor acts like a wire; an inductor acts like a broken wire.
After a long time, a capacitor acts like a broken wire, and inductor acts like a wire.

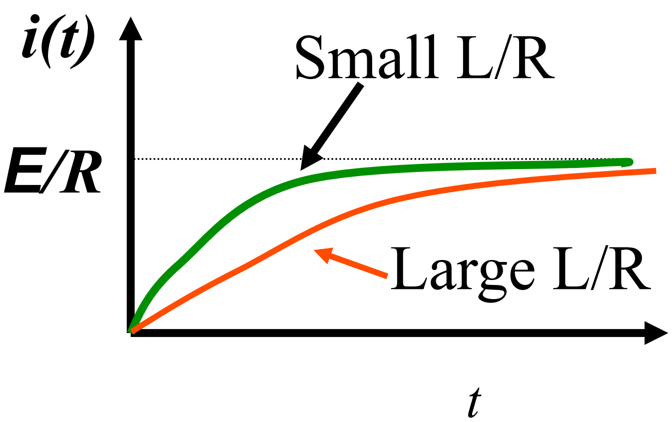
"Charging" an inductor

- How does the current in the circuit change with time?



$$-iR + \mathcal{E} - L\frac{di}{dt} = 0$$

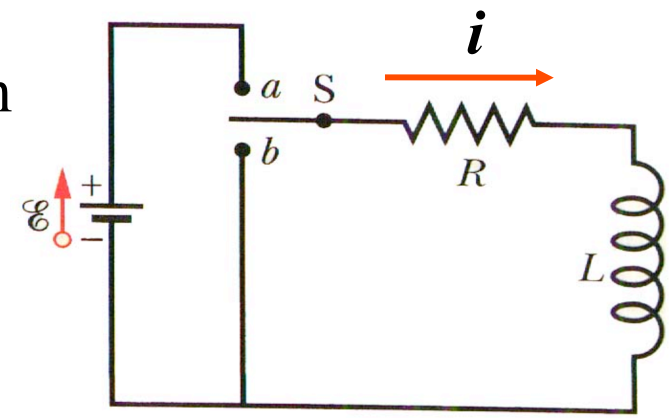
$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$



"Time constant" of RL circuit = L/R

“Discharging” an inductor

The switch is in a for a long time, until the inductor is charged. Then, the switch is closed to b.

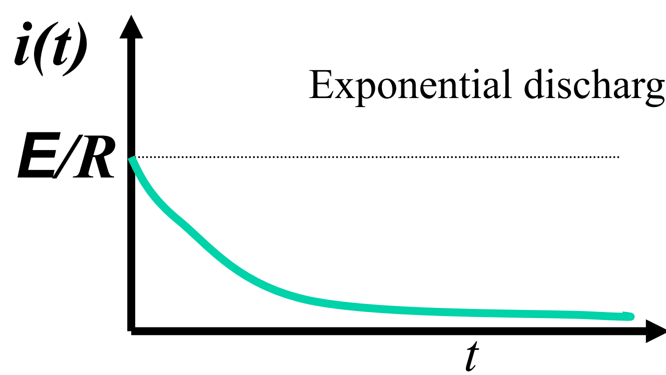


What is the current in the circuit?

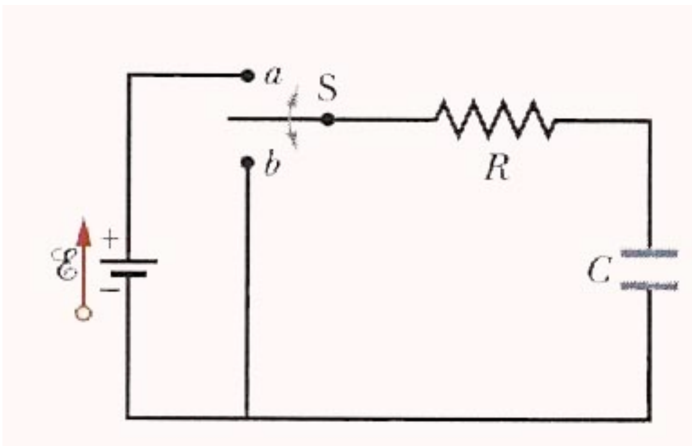
Loop rule around the new circuit:

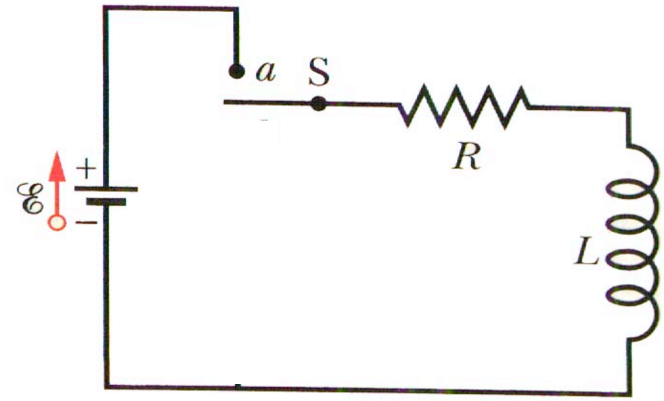
$$iR + L\frac{di}{dt} = 0$$

$$i = \frac{E}{R}e^{-\frac{Rt}{L}}$$



RL circuits





In an RC circuit, while charging, Q = CV and the loop rule mean:

- charge increases from 0 to CE
- current decreases from E/R to 0
- voltage across capacitor increases from 0 to E

In an RL circuit, while "charging" (rising current), emf = Ldi/dt and the loop rule mean:

- magnetic field increases from 0 to B
- current increases from 0 to E/R
- voltage across inductor decreases from −E to 0

Summary

- Notion of electric potential **does not work** for electric fields produced by induction

- **Inductance** of a solenoid

$$\Phi_B = NAB = Li$$

- SI unit **henry**

- **Self induction**: an EMF appears in any coil in which the current is changing: $EMF = -L\dfrac{di}{dt}$

- **Direction** of self-induced EMF from Lenz's law
- "Charging" an inductor: $i = \dfrac{\mathcal{E}}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

- "Discharging" an inductor: $i = \dfrac{\mathcal{E}}{R}e^{-\frac{Rt}{L}}$