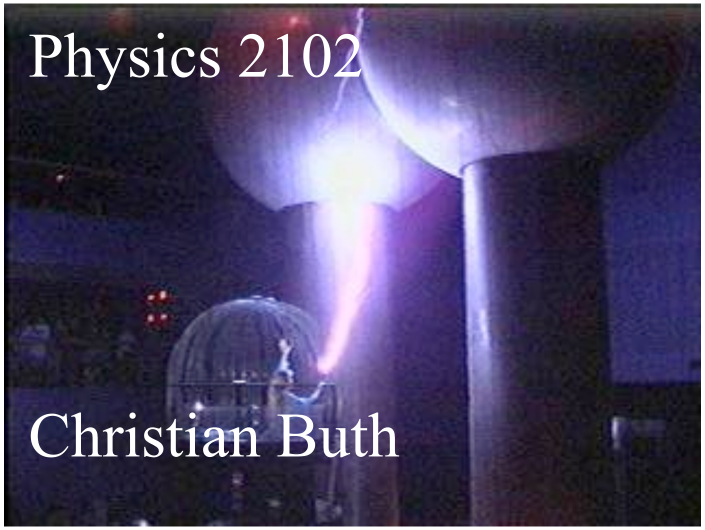


Physics 2102
Lecture 27
Alternative Faraday's Law

03/20/2009

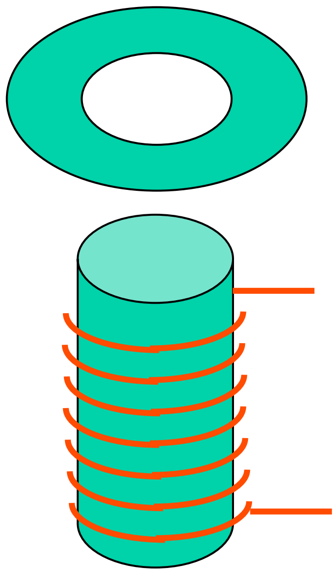
Nikola Tesla

Review

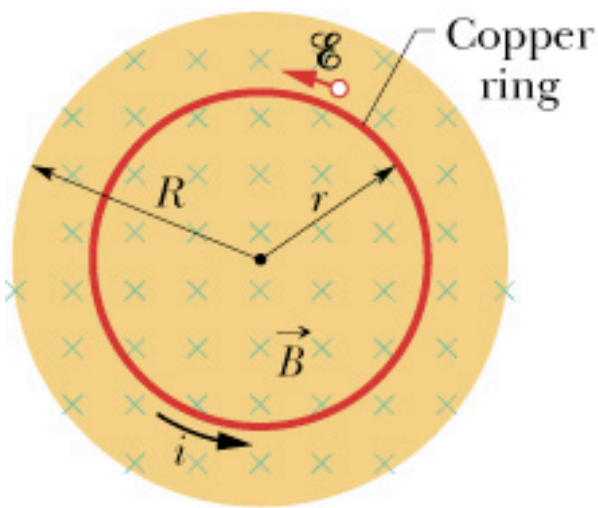
- Lenz's rule different formulation of **energy conservation**
- Thermal energy and mechanical energy are **equal** when pulling a conducting loop through a magnetic field
- A pendulum in magnetic fields is slowed down due to induced **eddy currents**

Example: Eddy Currents

- A non-magnetic (e.g. copper, aluminum) ring is placed near a solenoid.
- What happens if:
  - There is a steady current in the solenoid?
  - The current in the solenoid is suddenly changed?
  - The ring has a “cut” in it?



Induced Electric Fields



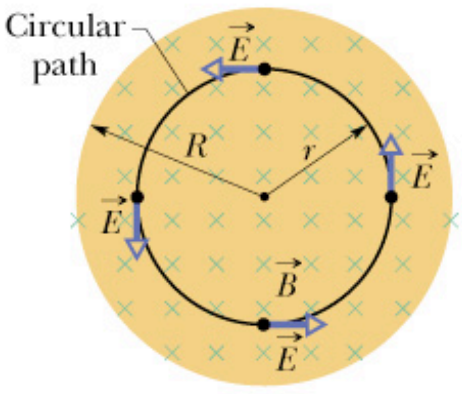
(a)

- Copper ring in **uniform** magnetic field
- Field **increases** with time
- Induces **counterclockwise** current
- Ergo: **electric field** is induced for current to occur
- **Another form** of Faraday's law:

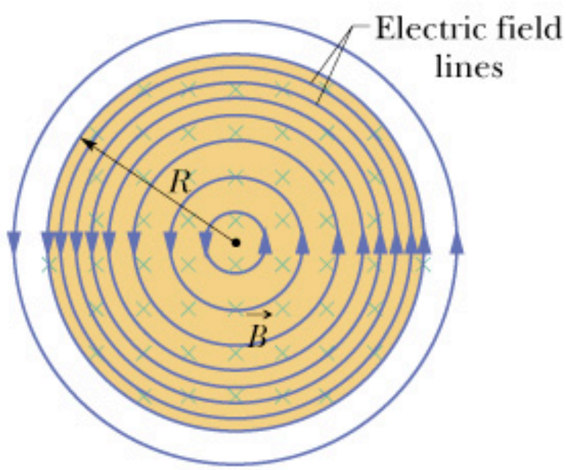
A changing magnetic field produces an electric field.

- Even in **absence** of ring, an electric field exists

Faraday's Law



(b)



(c)

- Copper ring **removed**
- emf **along path**:

$$EMF = \oint \vec{E} \cdot d\vec{s}$$

- From Faraday's law:

$$EMF = -\frac{d\Phi_B}{dt}$$

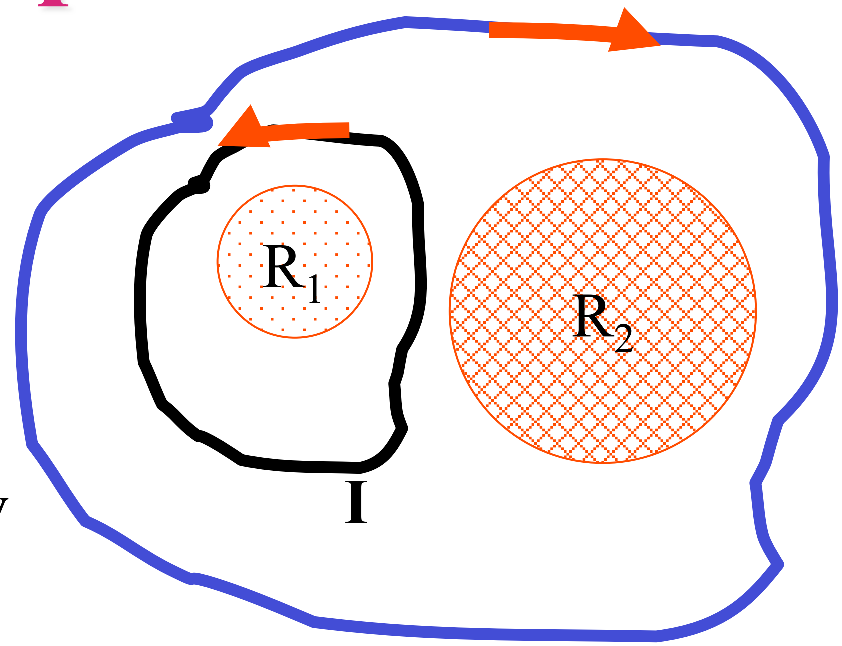
- **General form** of Faraday's law:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Example

- The figure shows two circular regions R_1 & R_2 with radii $r_1 = 1m$ & $r_2 = 2m$. In R_1, the magnetic field B_1 points out of the page. In R_2, the magnetic field B_2 points into the page
- Both fields are uniform and are DECREASING at the SAME steady rate = 1 T/s.
- Calculate the "Faraday" integral

$\oint_C \vec{E} \cdot d\vec{s}$ for the two paths shown



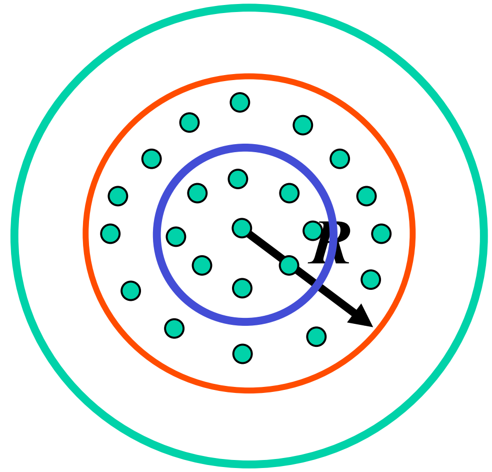
Path I: $$\oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} = -(\pi r_1^{\,2})(-1T/s) = +3.14V$$

Path II:

$$\oint_C \vec{E} \cdot d\vec{s} = -\left[-(\pi r_1^{\,2})(-1T/s) + (\pi r_2^{\,2})(-1T/s)\right] = +9.42V$$

Example

- A long solenoid has a circular cross-section of radius ***R***.
- The current through the solenoid is **increasing** at a steady rate ***di/dt***.
- Compute the variation of the electric field as a function of the distance ***r*** from the axis of the solenoid.



First, let's look at ***r < R:***

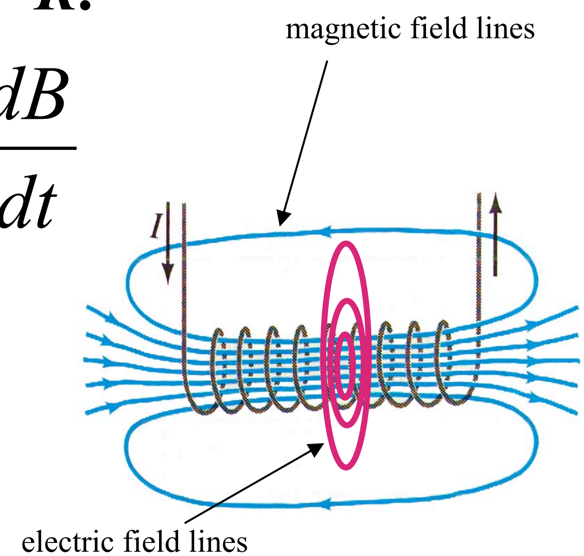
$$|E|(2\pi r) = (\pi r^2)\frac{dB}{dt}$$

$$E = \frac{r}{2}\frac{dB}{dt}$$

Next, let's look at ***r > R:***

$$|E|(2\pi r) = (\pi R^2)\frac{dB}{dt}$$

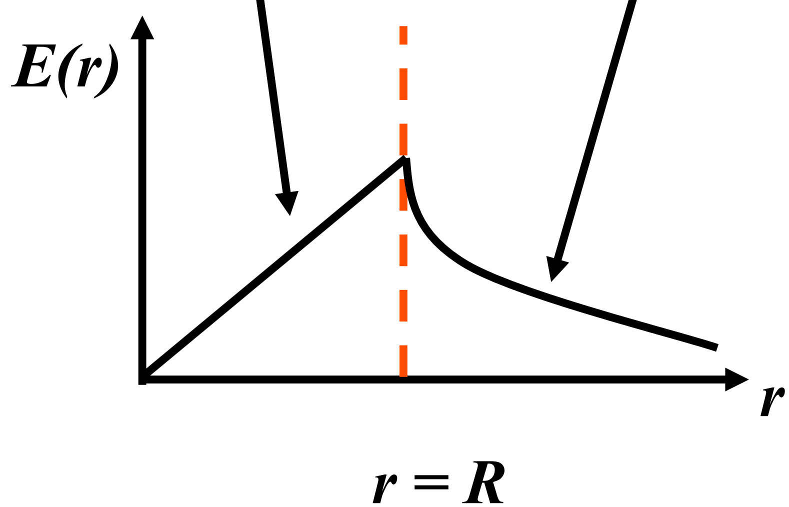
$$E = \frac{R^2}{2r}\frac{dB}{dt}$$



Example (continued)

$$E = \frac{r}{2}\frac{dB}{dt}$$

$$E = \frac{R^2}{2r}\frac{dB}{dt}$$



Summary

- Alternative version of **Faradays' law**:

A changing magnetic field produces an electric field.

$$\oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

- The **electric field** inside / outside a solenoid with increasing current

$$E = \frac{r}{2}\frac{dB}{dt}$$

$$E = \frac{R^2}{2r}\frac{dB}{dt}$$