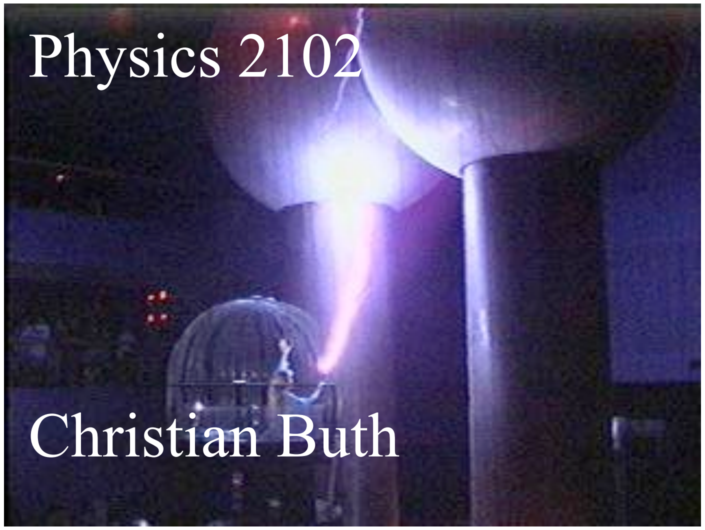


Physics 2102
Lecture 25

Induction 1

03/16/2009

Review

- Inside a wire with uniform current density:

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

- **Solenoid** is tightly wound helical wire $B = \mu_0 n i$
- **Toroid** is a **doughnut** shaped coil

$$B = \frac{\mu_o N i}{2\pi r}$$

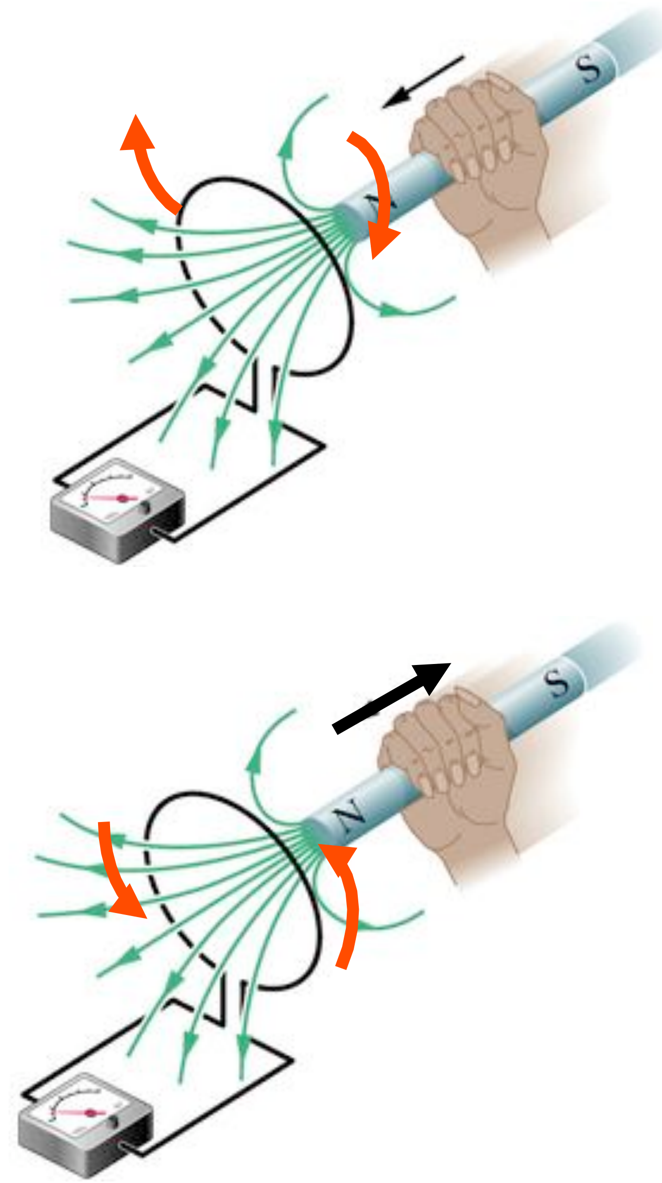
- Magnetic field of circular loop is a magnetic dipole

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{(R^2 + z^2)^{3/2}} \approx \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$$

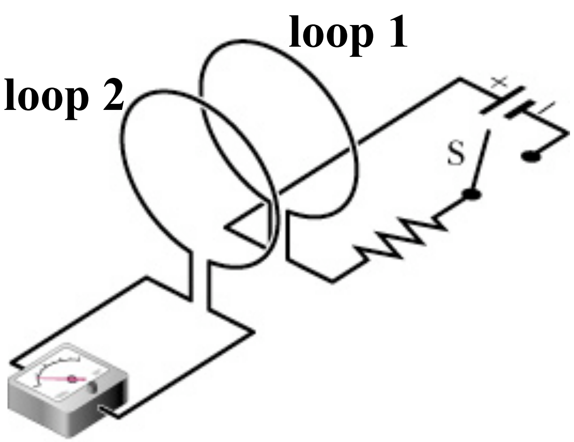
- **Magnetic dipole moment** of magnitude $\mu = NiA$

Faraday's Experiments 1

- When the North pole approaches the loop, the ammeter shows a **current**
- **Faster** motion yields larger current
- **Reverse** direction of motion reverses the direction of the **induced current**
- This is due to an **induced emf**



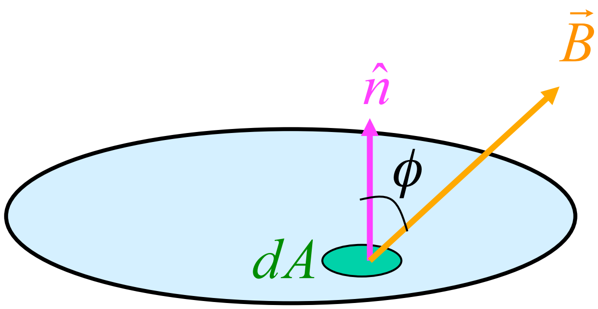
Faraday's Experiments 2



- Current in loop 2 when **switch** is opened or closed
- Only the **change** of the magnetic field matters

An emf is induced in a loop when the number of magnetic field lines that pass through the loop is changing.

Magnetic Flux through Surface



- **Divide** complex surface into elements of area d***A***
- **Magnetic flux**:

$$\Phi_B = \int \vec{B} \cdot \mathrm{d}\vec{A} = \int B\,\mathrm{d}A \cos\phi$$

- **SI unit** of flux 1 T m^2 = 1 Weber = 1 Wb

Faraday's Law of Induction

- The **induced emf** in a conductive loop is

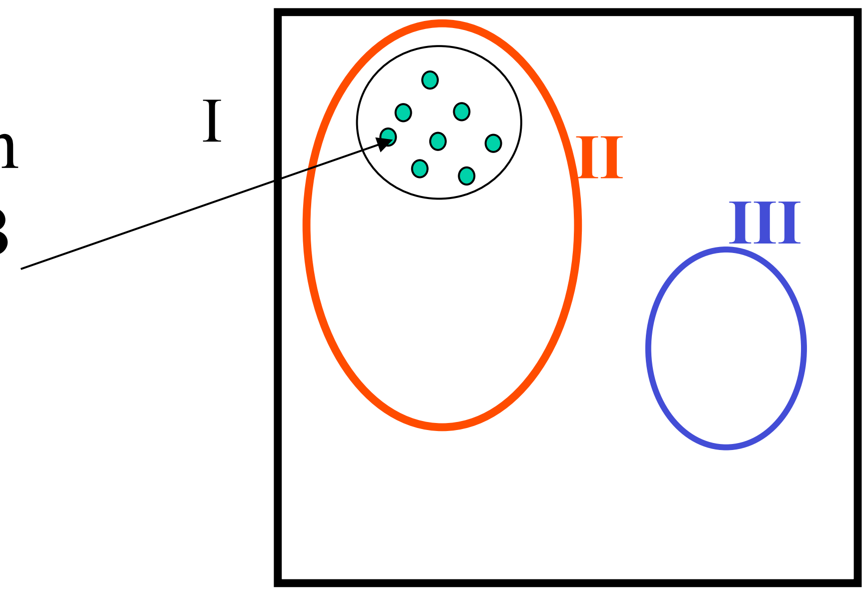
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

The magnitude of the emf $\mathcal{E}$ induced in a conductive loop is equal to the rate at which the magnetic flux Φ_B through the loop changes with time.

- **Changing** the magnetic flux:
  - Change the magnetic field
  - Change the area of the coil or the part in the field
  - Change the angle ϕ

Example

- 3 loops are shown.
- B = 0 everywhere except in the circular region where B is uniform, pointing out of the page and is **increasing at a steady rate.**
- Rank the 3 loops in order of increasing induced EMF.
  - (a) III, II, I
  - (b) III, (I & II are same)
  - (c) ALL SAME.

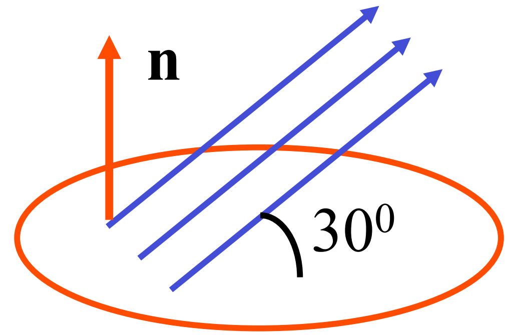


• Just look at the rate of change of ENCLOSED flux

• III encloses no flux and it does not change.

• I and II enclose same flux and it changes at same rate.

Example



- A closed loop of wire encloses an area of 1 m^2 in which a uniform magnetic field exists at 30^0 to the PLANE of the loop. The magnetic field is DECREASING at a rate of 1T/s. The resistance of the wire is 10 Ω.
- What is the induced current?

$$\Phi_B = \int_S \vec{B} \cdot \hat{n} \, dA$$

$$= BA\cos(60^0) = \frac{BA}{2}$$

$$|EMF| = \frac{d\Phi_B}{dt} = \frac{A}{2}\frac{dB}{dt}$$

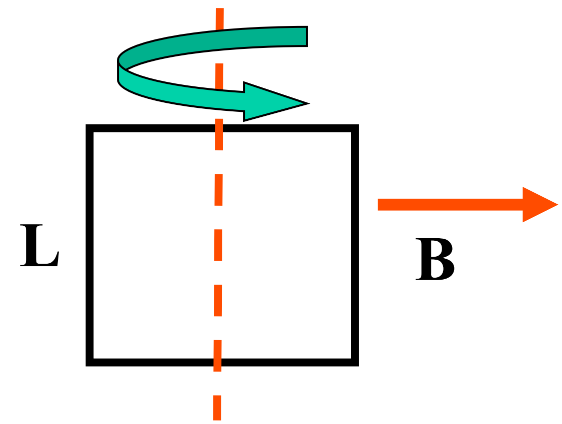
$$i = \frac{EMF}{R} = \frac{A}{2R}\frac{dB}{dt}$$

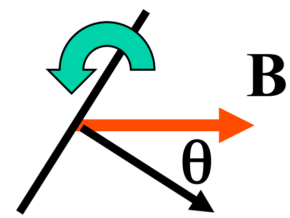
$$i = \frac{(1m^2)}{2(10\Omega)}(1T/s) = 0.05A$$

Is it
…clockwise or
…counterclockwise?

The Generator

- A square loop of wire of side ***L*** is rotated at a uniform frequency ***f*** in the presence of a uniform magnetic field ***B*** as shown.
- Describe the EMF induced in the loop.





$$\Phi_B = \int_S \vec{B} \cdot \hat{n} dA$$

$$= BL^2 \cos(\theta)$$

$$EMF = -\frac{d\Phi_B}{dt} = BL^2 \frac{d\theta}{dt} \sin(\theta) = BL^2 (2\pi f) \sin(2\pi f t)$$

Lenz's Rule

- Minus sign in Faraday's law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- To predict direction of induced current
- Negative sign follows from **Lenz's rule**:

An induced current has a direction such that the magnetic field due to the induced current opposes the change in the magnetic flux that induces the current.

Summary

- Magnetic flux: $\Phi_B = \int \vec{B} \cdot d\vec{A}$
- Faradays' law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$

An emf is induced in a loop when the number of magnetic field lines that pass through the loop is changing.

- Negative sign in Faradays' law from **Lenz rule**:

An induced current has a direction such that the magnetic field due to the induced current opposes the change in the magnetic flux that induces the current.