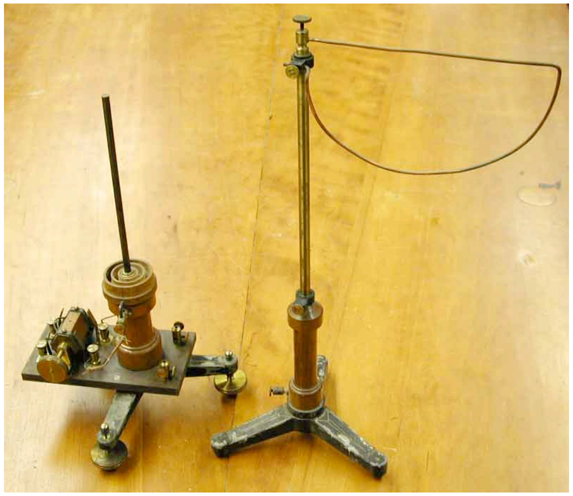
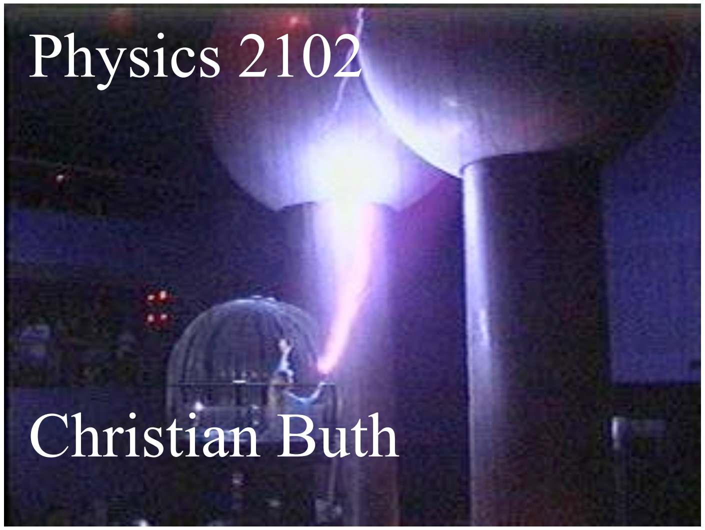
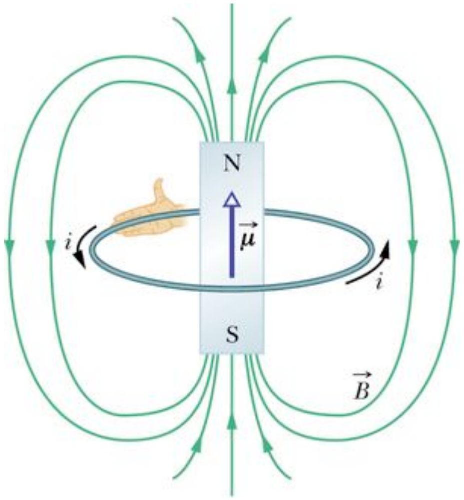




Physics 2102
Lecture 24
Ampere's law 2

Version: 03/13/2009



André Marie Ampère
(1775 – 1836)

Review

- Wires carrying currents produce forces on each other: **parallel currents attract**, antiparallel currents repel
- Force between wires

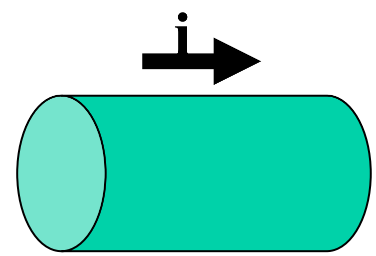
$$F_{21} = L I_2 B_1 = \frac{\mu_0 L I_1 I_2}{2\pi a}$$

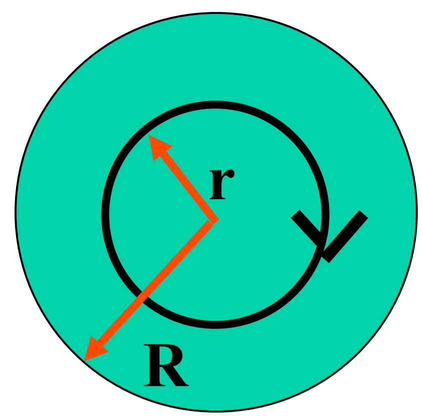
- **Ampere's law** analog to Gauss' law for electric fields:

> The line integral $\oint \vec{B} \cdot d\vec{s}$ of the magnetic field $\vec{B}$ along any closed path is equal to the total current enclosed inside the path multiplied by μ_0.

Ampere's Law: Example 2

- Infinitely long cylindrical wire of finite radius **R** carries a total current **i** with uniform current density
- Compute the magnetic field at a distance **r** from cylinder axis for:
  - r < a (inside the wire)
  - r > a (outside the wire)



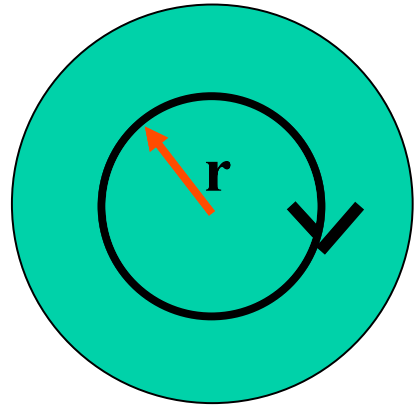


Current into page, circular field lines

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i$$

Ampere's Law: Example 2 (cont)

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i$$



Current into page, field tangent to the closed amperian loop

$$B(2\pi r) = \mu_0 i_{enclosed}$$

$$B = \frac{\mu_0 i_{enclosed}}{2\pi r}$$

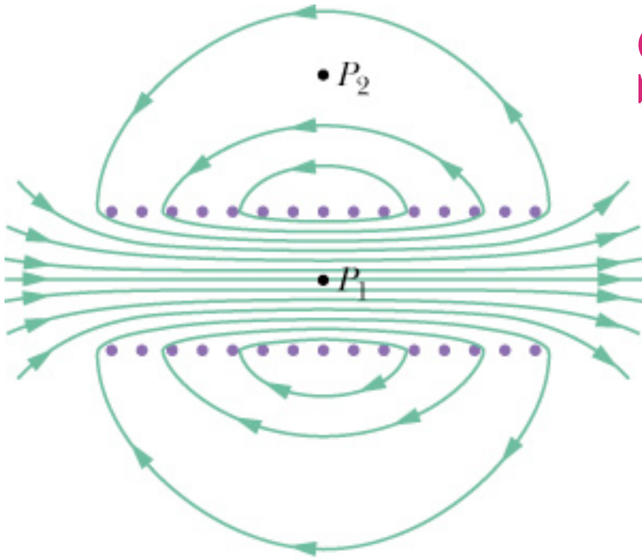
$$i_{enclosed} = J(\pi r^2) = \frac{i}{\pi R^2}\pi r^2 = i\frac{r^2}{R^2}$$

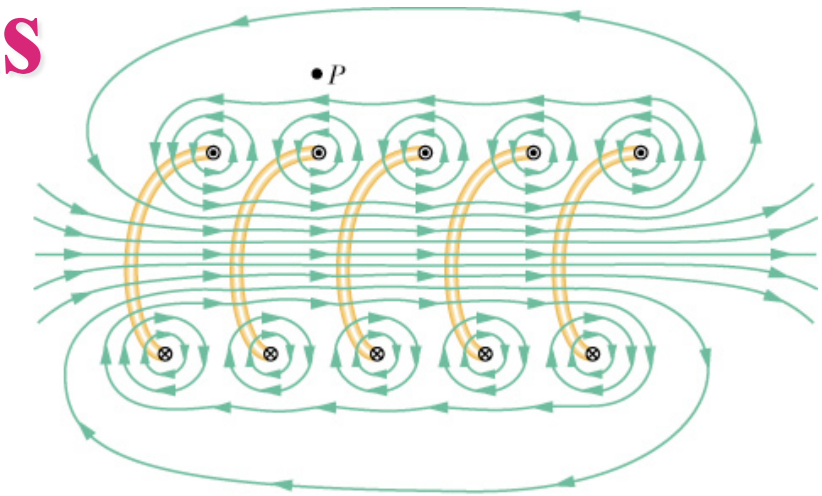
$$B = \frac{\mu_0 i r}{2\pi R^2}$$

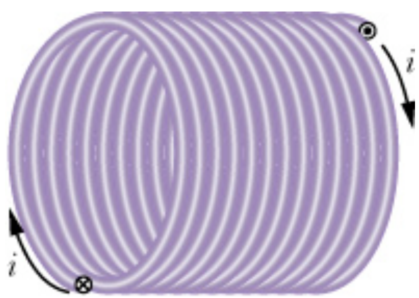
For r < R

For r > R, $i_{enc} = i$, so $B = \mu_0 i / 2\pi R$

Solenoids

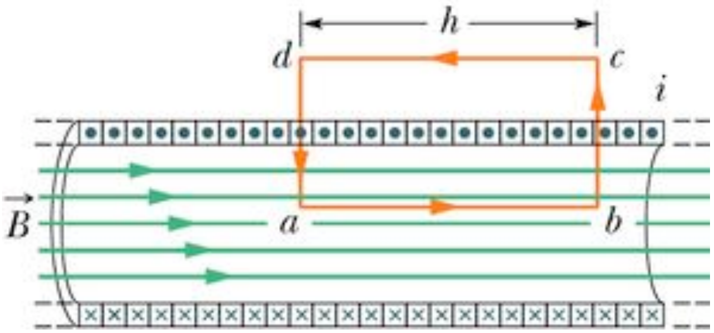






- **Solenoid** is tightly wound helical wire
- Coil length **much** larger than coil diameter
- Magnetic field in solenoid **uniform** and **parallel** to axis
- Outside away from ends ***B*** almost **zero**
- Sense of ***B***: curl fingers from **right hand** along direction of current; thumb points along ***B***

Magnetic Field of Solenoid



- ***B*** from **Ampere's law**
- *n* number of **turns** per unit length
- Outside solenoid and perpendicular to its axis ***B*** d***s*** = **0**
- Magnetic field **independent** of the solenoid's radius

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

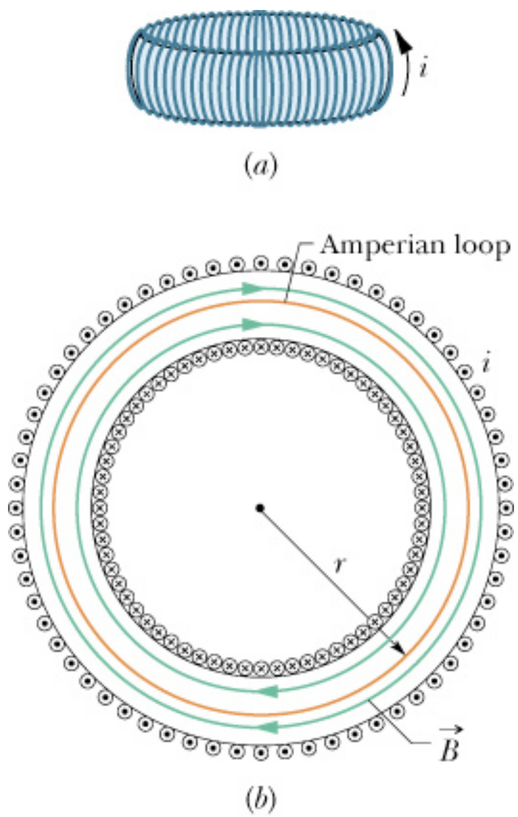
$$\oint \vec{B} \bullet d\vec{s} = 0 + B\,h + 0 + 0$$

$$i_{enc} = iN_h = i(N/L)h = inh$$

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

$$\Rightarrow Bh = \mu_0 inh \Rightarrow B = \mu_0 in$$

Magnetic Field of Toroid



- Magnetic field in **circles** concentric with toroid's center
- Sense of ***B***: curl fingers from **right hand** along direction of current; thumb points along ***B***
- Magnetic field outside toroid almost **zero**

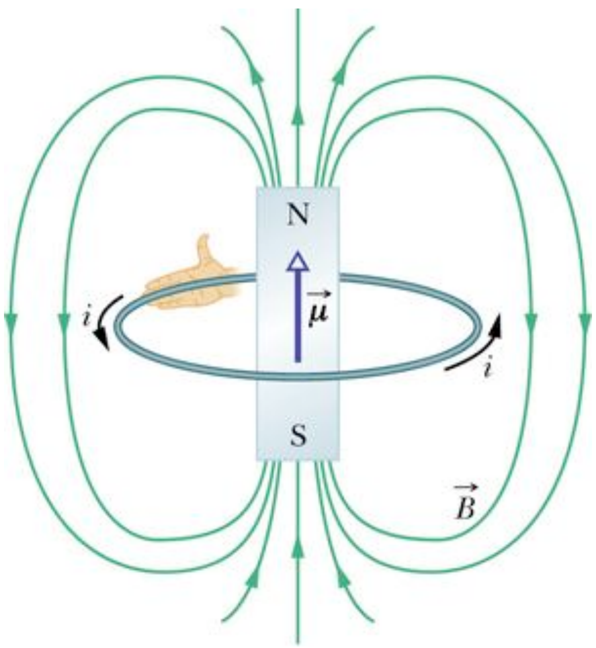
$$\oint \vec{B} \bullet d\vec{s} = B \oint ds = \mu_0 i_{enc}$$

$$\Rightarrow 2\pi r B = \mu_0 N i$$

$$\Rightarrow B = \frac{\mu_0 N i}{2\pi r}$$

Magnetic Field of a Magnetic Dipole

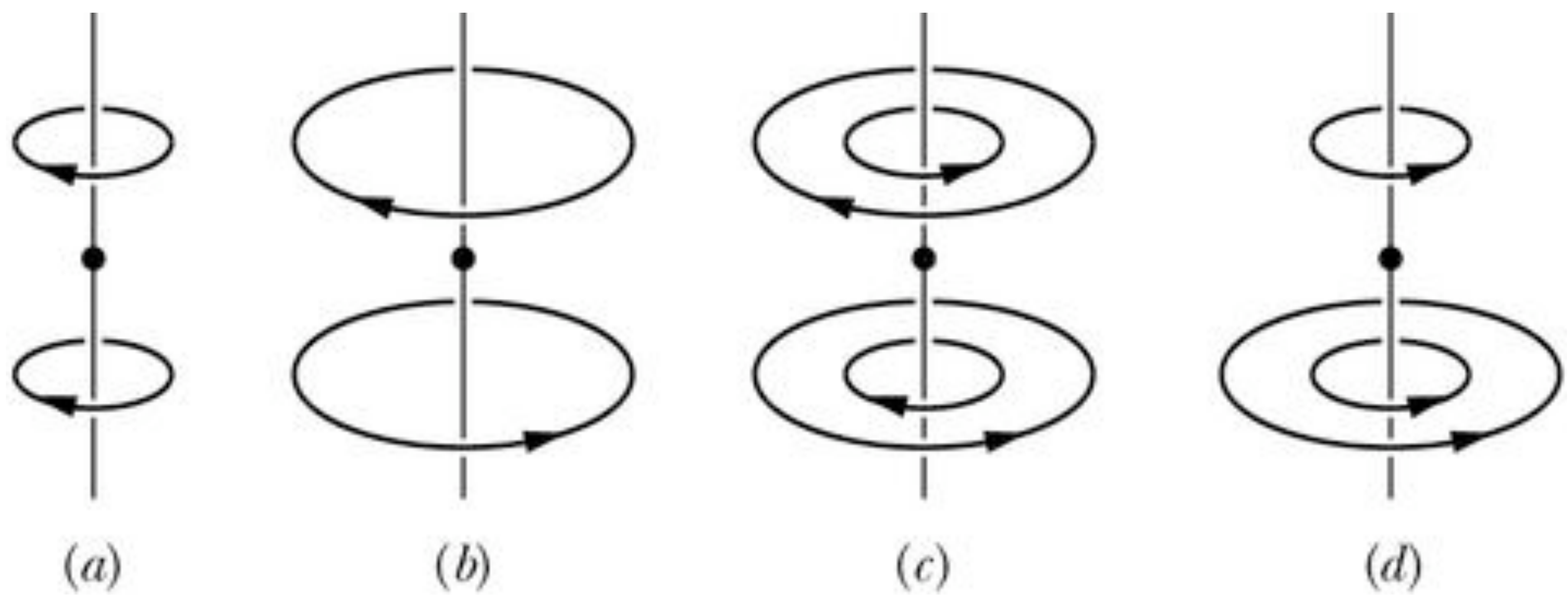
- **Circular loop** with electrical current is a magnetic dipole
- **Magnetic dipole moment** of magnitude $\mu = NiA$
- Dipole moment points in **South-North** direction
- Sense of $\boldsymbol{B}$: curl fingers from **right hand** along direction of current; thumb points along $\boldsymbol{B}$



$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{(R^2 + z^2)^{3/2}} \approx \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$$

Example

- All loops in the figure have radius *r* or 2*r*
- Which of these arrangements produce the **largest magnetic field** at the point indicated?



Summary

- Inside a wire with uniform current density:

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

- **Solenoid** is tightly wound helical wire $B = \mu_0 n i$
- **Toroid** is a **doughnut** shaped coil

$$B = \frac{\mu_o N i}{2\pi r}$$

- Magnetic field of circular loop is a magnetic dipole

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{(R^2 + z^2)^{3/2}} \approx \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$$

- **Magnetic dipole moment** of magnitude $\mu = NiA$