

Physics 2102

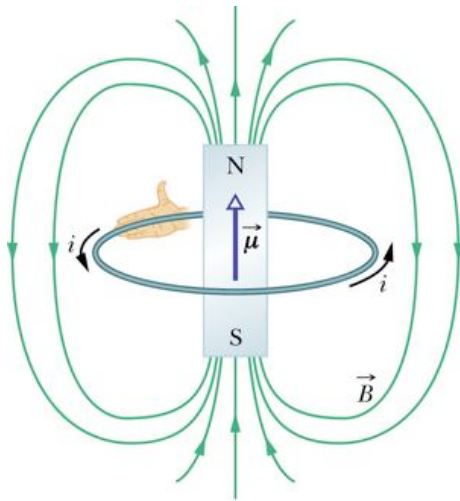
Lecture 23

Ampere's law 1

Version: 03/11/2009



André Marie Ampère
(1775 – 1836)



Review

- Magnetic fields from currents from **Biot-Savart's law**:

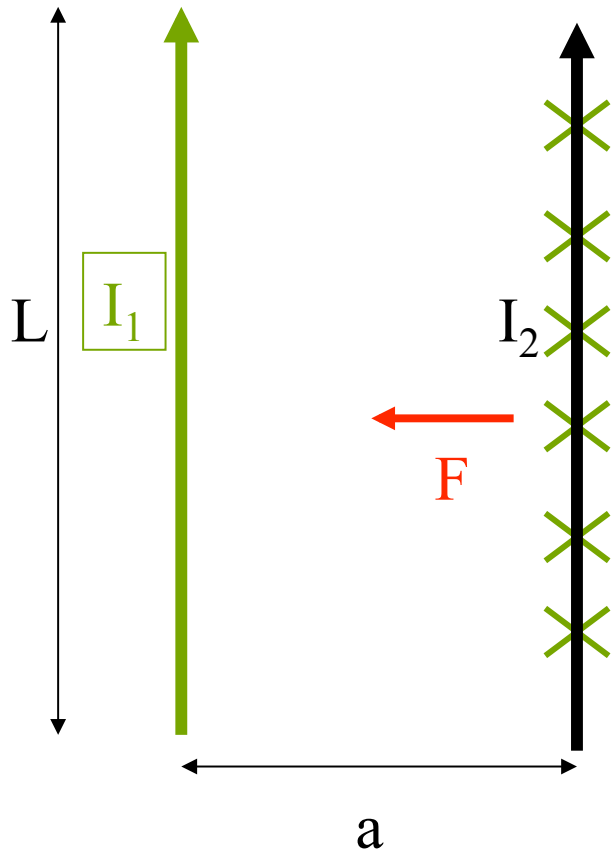
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{s} \times \vec{r}}{r^3}$$

- Two **right-hand rules** for direction of magnetic field
- **Straight currents** produce circular magnetic fields:
- Current through **circular arc** produce magnetic field at center:

$$B = \mu_0 i / 2\pi r$$

$$B = \mu_0 i \Phi / 4\pi r$$

Forces between wires



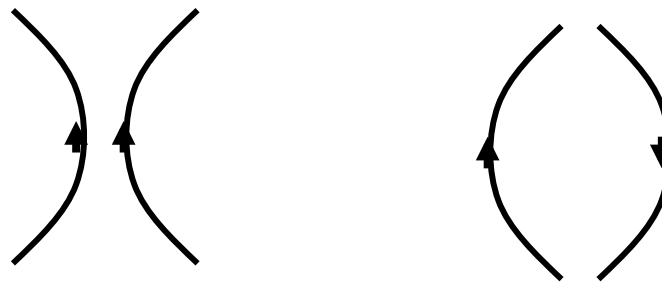
Magnetic field due to wire 1
where the wire 2 is,

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

Force on wire 2 due to this field,

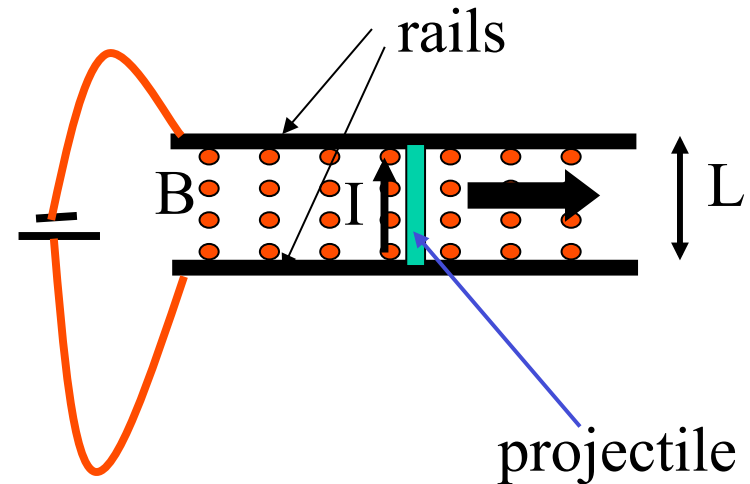
$$\vec{F}_{12} = i_2 \vec{L} \times \vec{B}_1$$

$$F_{21} = L I_2 B_1 = \frac{\mu_0 L I_1 I_2}{2\pi a}$$



The Rail Gun

- Conducting projectile of length 2cm, mass 10g carries constant current 100A between two rails
- Magnetic field $B = 100\text{T}$ points outward
- Assuming the projectile starts from rest at $t = 0$, what is its speed after a time $t = 1\text{s}$?



- Force on projectile: $F = iLB$ (from $F = i\mathbf{L} \times \mathbf{B}$)
- Acceleration: $a = iLB/m$ (from $F = ma$)
- $v(t) = iLBt/m$ (from $v = v_0 + at$)

$$= (100\text{A})(0.02\text{m})(100\text{T})(1\text{s})/(0.01\text{kg}) = 2000\text{m/s}$$

$$= 4,473\text{mph} = \text{MACH } 8!$$

Rail guns in the “Eraser” movie

“Rail guns are hyper-velocity weapons that shoot aluminum or clay rounds at just below the speed of light. In our film, we've taken existing stealth technology one step further and given them an X-ray scope sighting system,” notes director Russell. “These guns represent a whole new technology in weaponry that is still in its infancy, though a large-scale version exists in limited numbers on

battleships and tanks. They have incredible range. They can pierce three-foot thick cement walls and then knock a canary off a tin can with absolute accuracy. In our film, one contractor has finally developed an assault-sized rail gun. We researched this quite a bit, and the technology is really just around the corner, which is one of the exciting parts of the story.”



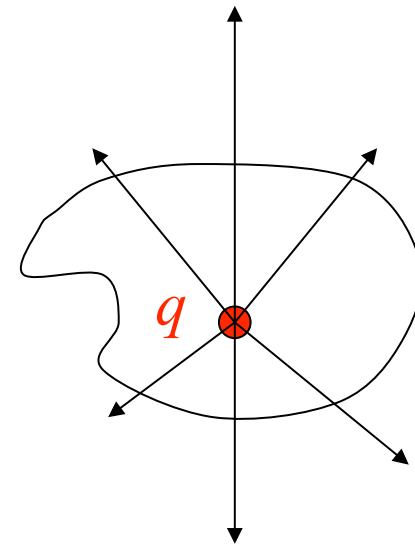
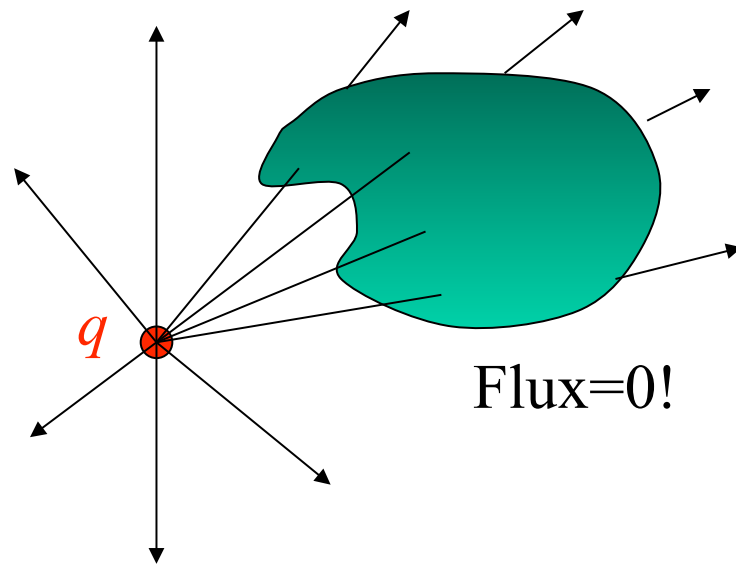
Warner Bros., production notes, 1996.

<http://movies.warnerbros.com/eraser/cmp/prodnotes.html#tech>

Also: INSULTINGLY STUPID MOVIE PHYSICS: <http://www.intuitor.com/moviephysics/>

Remember Gauss' law!

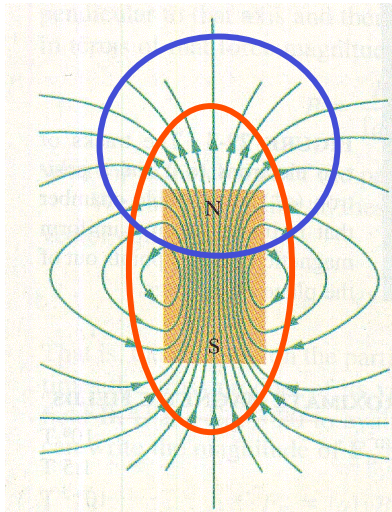
Given an **arbitrary** closed surface, the electric flux through it is proportional to the charge enclosed by the surface



$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

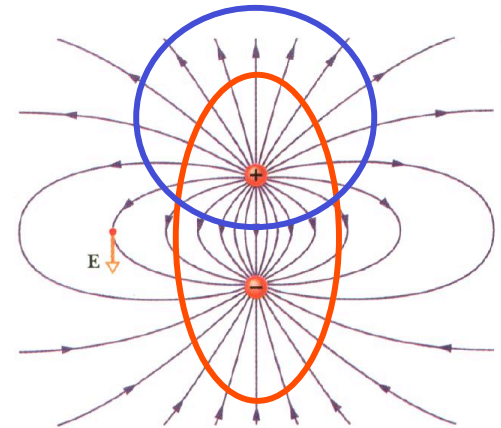
Analog for Magnetic Fields

No isolated magnetic poles! The magnetic flux through any closed “Gaussian surface” will be ZERO. This is one of the four “Maxwell’s equations”



$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

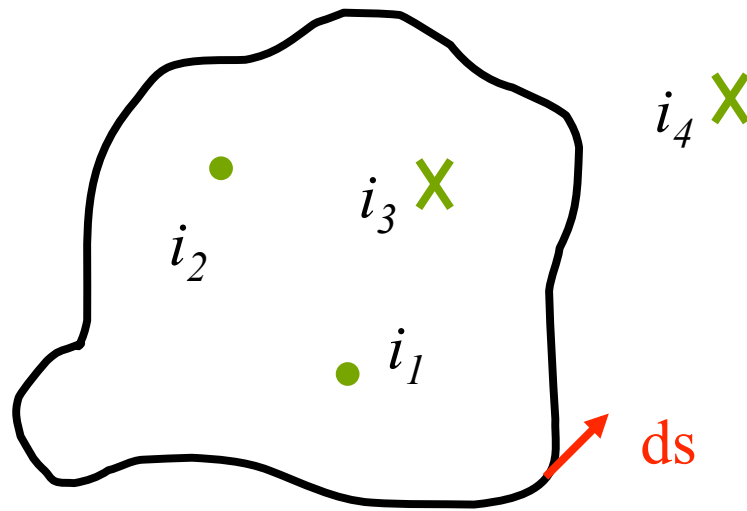


Ampere's law



$$\oint_{\text{loop}} \vec{B} \cdot d\vec{s} = \mu_0 i$$

The circulation of \vec{B} (the integral of \vec{B} scalar ds) along an imaginary **closed** “**Amperian**” loop is proportional to the **net amount of current** traversing the loop



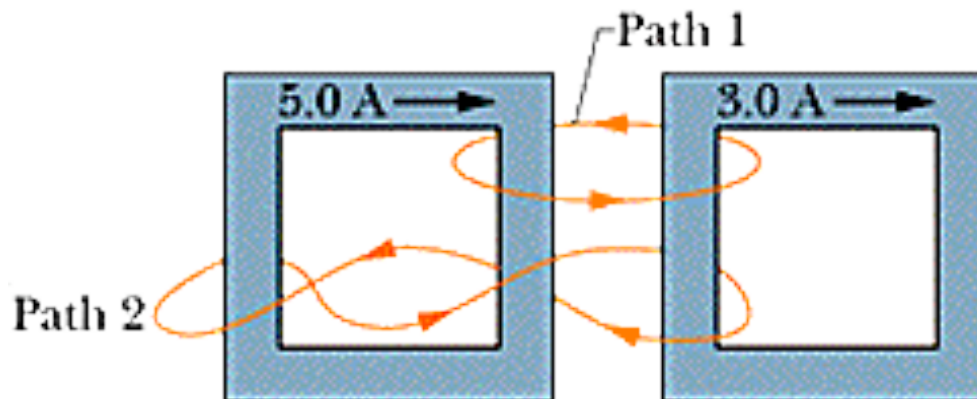
$$\oint_{\text{loop}} \vec{B} \cdot d\vec{s} = \mu_0 (i_1 + i_2 - i_3)$$

Thumb rule for sign; ignore i_4

As was the case for Gauss' law, if you have a lot of **symmetry**, knowing the circulation of \vec{B} allows you to find \vec{B}

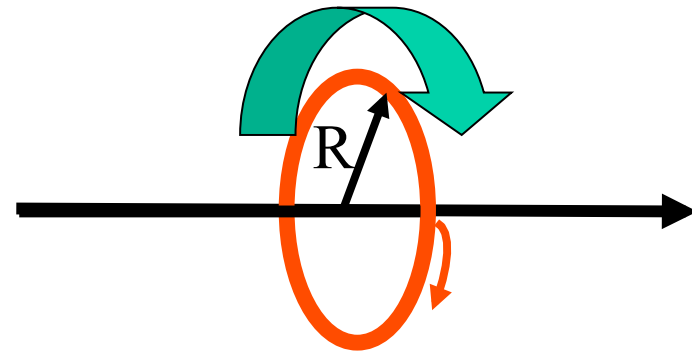
Sample Problem

- Two square conducting loops carry currents of 5.0 and 3.0 A as shown in Fig. 30-60. What's the value of $\int \mathbf{B} \cdot d\mathbf{s}$ through each of the paths shown?



Ampere's Law: Example 1

- Infinitely long **straight wire** with current i
- Symmetry: magnetic field consists of **circular loops** centered around wire
- Choose a circular loop C -- B is **tangential** to the loop everywhere!
- B and ds parallel (Go round loop in same direction as B field lines!)



$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i$$

$$\oint_C B ds = B(2\pi R) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi R}$$

Summary

- Wires carrying currents produce forces on each other: **parallel currents attract**, antiparallel currents repel

- Force between wires

$$F_{21} = L I_2 B_1 = \frac{\mu_0 L I_1 I_2}{2\pi a}$$

- **Ampere's law** analog to Gauss' law for electric fields:

The line integral $\oint \vec{B} \cdot d\vec{s}$ of the magnetic field \vec{B} along any closed path is equal to the total current enclosed inside the path multiplied by μ_0 .