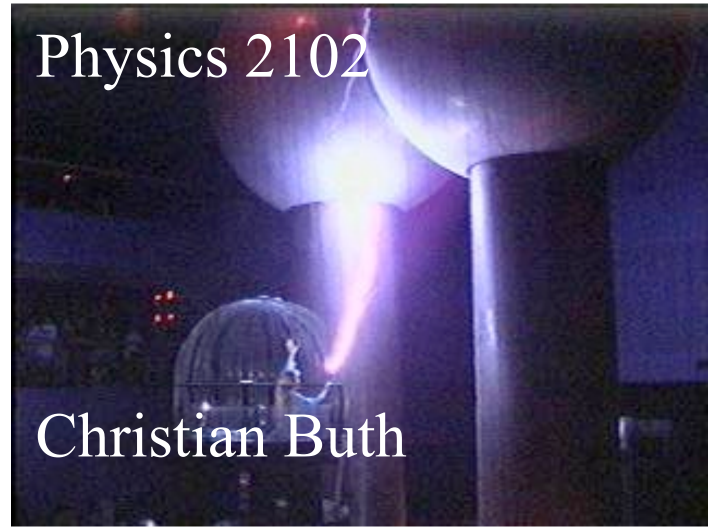
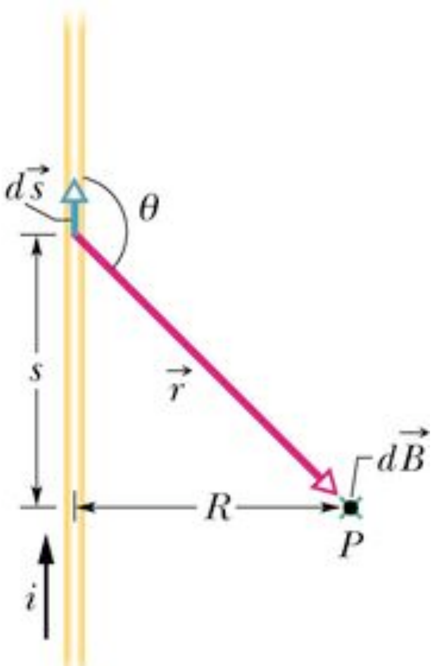


Physics 2102
Lecture 22

Version: 03/09/2009

Biot-Savart Law



Jean-Baptiste Biot
(1774-1862)

Felix Savart
(1791–1841)

Review

- **Cyclotrons** and **synchrotrons** to accelerate particles
- Wires carrying currents experience a force in a magnetic field
- For a straight wire: $\vec{F}_B = i\vec{L} \times \vec{B}$, generally: $\vec{F}_B = i\int d\vec{L} \times \vec{B}$
- **Current loop** is **magnetic dipole;** in uniform magnetic field

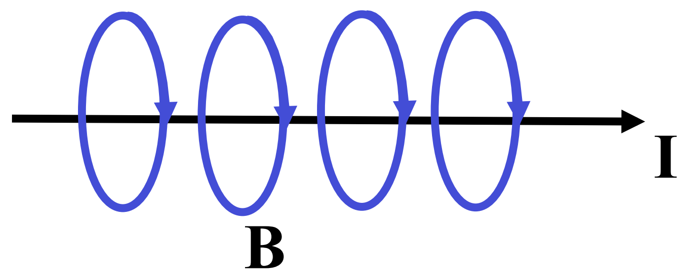
$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad \vec{\mu} = (NiA)\hat{n}$$

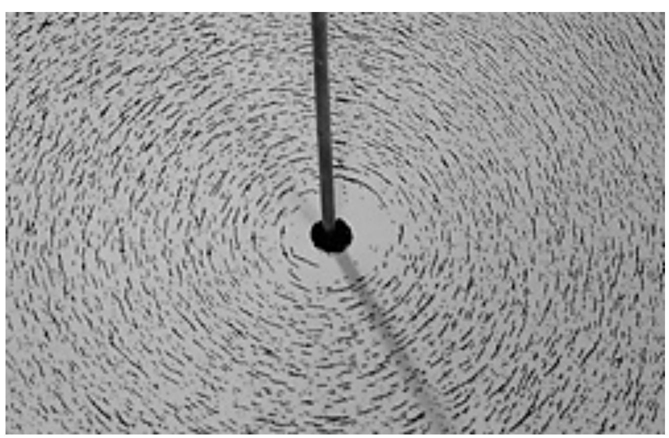
- **Right-hand rule** gives direction of moment
- **Magnetic potential energy** of magnetic dipole in uniform magnetic field

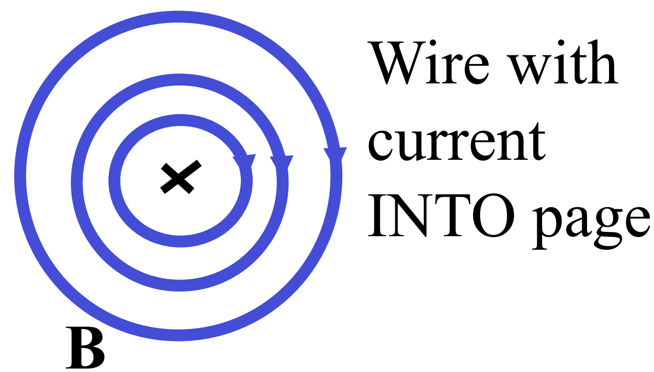
$$U = -\vec{\mu} \cdot \vec{B}$$

Electric Current:
A Source of Magnetic Field

- Observation: an electric current creates a magnetic field
- Simple experiment: hold a current-carrying wire near a compass needle!

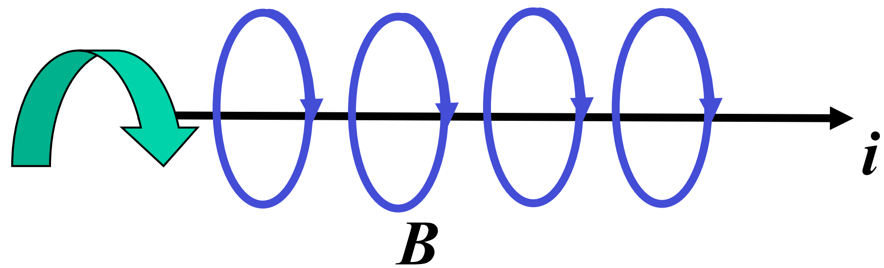


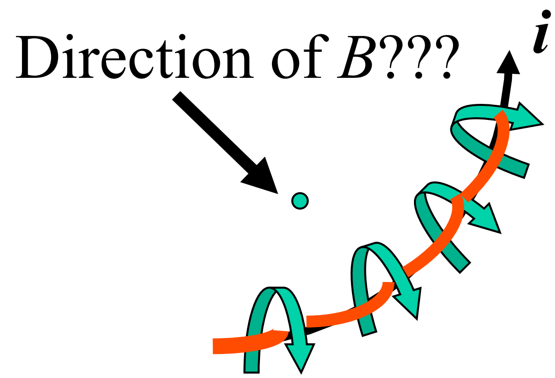




Yet Another Right Hand Rule!

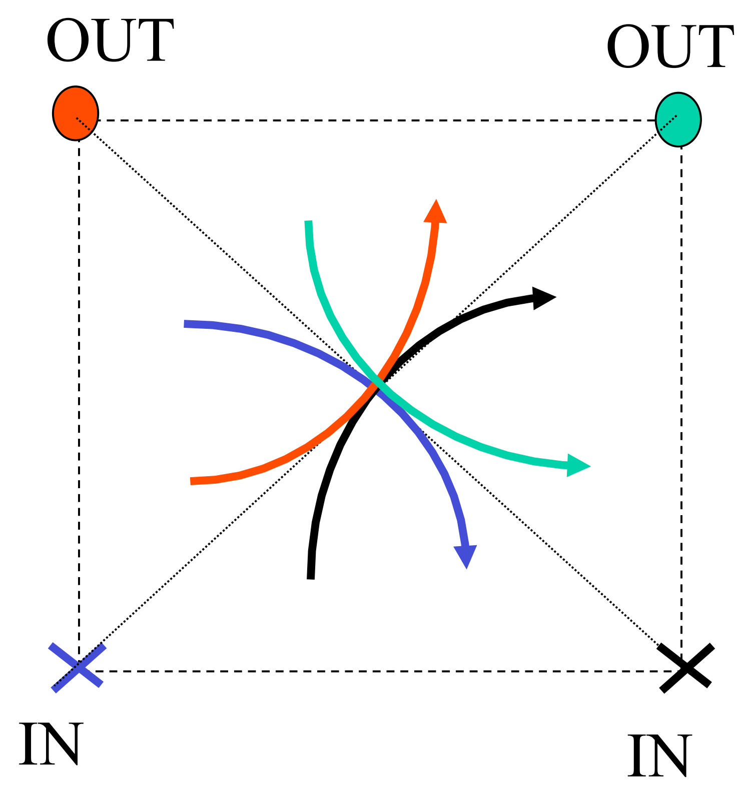
- Point your thumb along the direction of the current in a straight wire
- The magnetic field created by the current consists of circular loops directed along your curled fingers
- The magnetic field gets weaker with distance
- You can apply this to ANY straight wire (even a small differential element!)
- What if you have a curved wire? Break into small elements

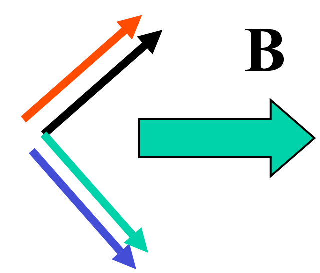




Superposition

- Magnetic fields (like electric fields) can be "superimposed" -- just do a vector sum of *B* from different sources
- The figure shows four wires located at the 4 corners of a square. They carry equal currents in directions indicated
- What is the direction of B at the center of the square?

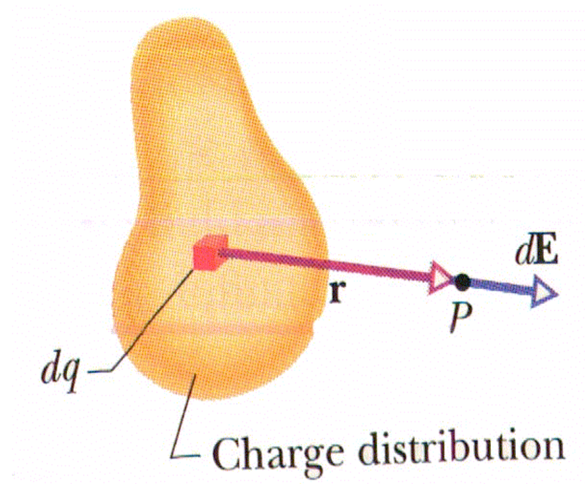




Comparison: electric field

When we computed the electric field due to charges we used **Coulomb's law**. If one had a large irregular object, one broke it into infinitesimal pieces and computed,

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$



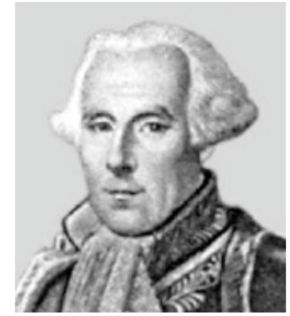
Which we write as,

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$$

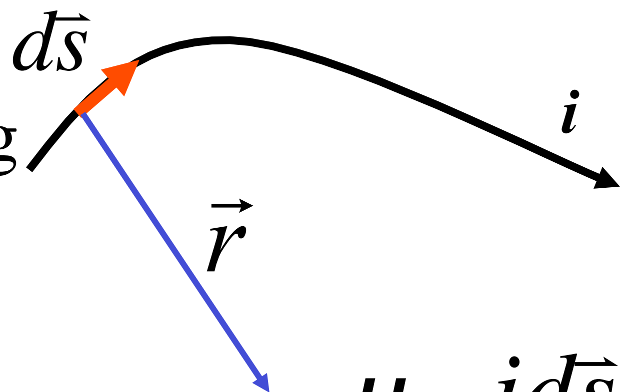
If you wish to compute the **magnetic field** due to a current in a wire, you use the law of **Biot and Savart**

Biot-Savart Law

Jean-Baptiste Biot (1774-1862)



Felix Savart (1791-1841)



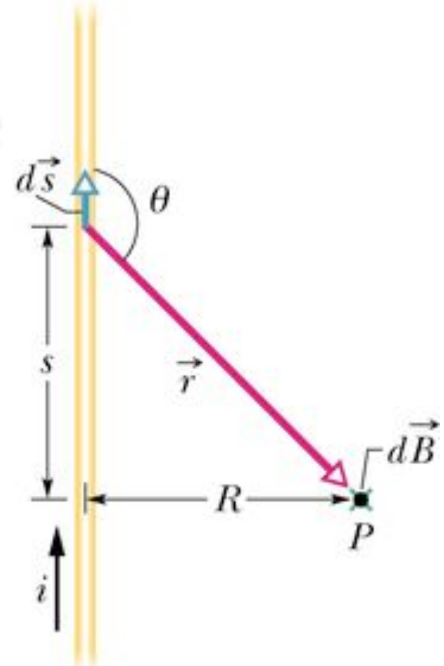
- Quantitative law for computing the magnetic field from any electric current
- Choose a differential element of wire of length ***ds*** and carrying a current ***i***
- The field ***dB*** from this element at a point located by the vector ***r*** is given by the Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3}$$

$\mu_0 = 4\pi x 10^{-7}$ Tm/A (permeability constant)

Compare with $d\vec{E} = \frac{dq}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$

Long Straight Wire



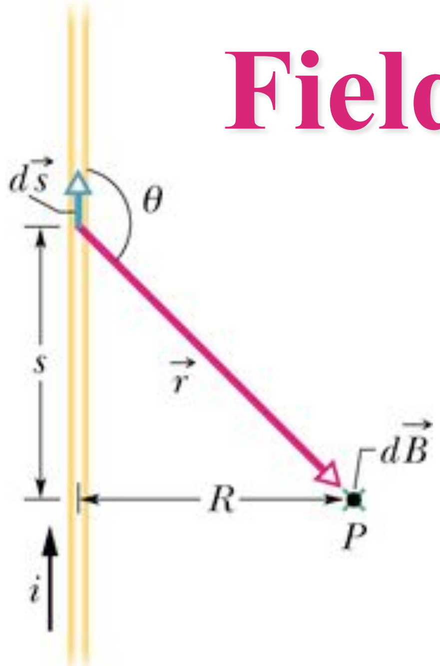
- An infinitely long straight wire carries a current ***i***
- Determine the magnetic field generated at a point located at a perpendicular distance ***R*** from the wire
- Choose an element d***s*** as shown
- Biot-Savart Law: ***dB*** points **into** the page
- Integrate over all such elements

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} \frac{i ds (r \sin\theta)}{r^3}$$

$$B = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{\infty} \frac{ds (r \sin\theta)}{r^3}$$

Field of a Long Straight Wire



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3} \qquad dB = \frac{\mu_0}{4\pi}\frac{i ds(r \sin\theta)}{r^3}$$

$$\sin\theta = R / r \qquad r = (s^2 + R^2)^{1/2}$$

$$B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{ds(r\sin\theta)}{r^3} = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{R ds}{\left(s^2 + R^2\right)^{3/2}}$$

$$= \frac{\mu_0 i}{2\pi}\int_{0}^{\infty}\frac{R ds}{\left(s^2 + R^2\right)^{3/2}}$$

$$= \frac{\mu_0 i R}{2\pi}\left[\frac{s}{R^2\left(s^2 + R^2\right)^{1/2}}\right]_0^{\infty} \qquad \boxed{= \frac{\mu_0 i}{2\pi R}}$$

Example: A Practical Matter

A power line carries a current of 500 A. What is the magnetic field in a house located 100 m away from the power line?

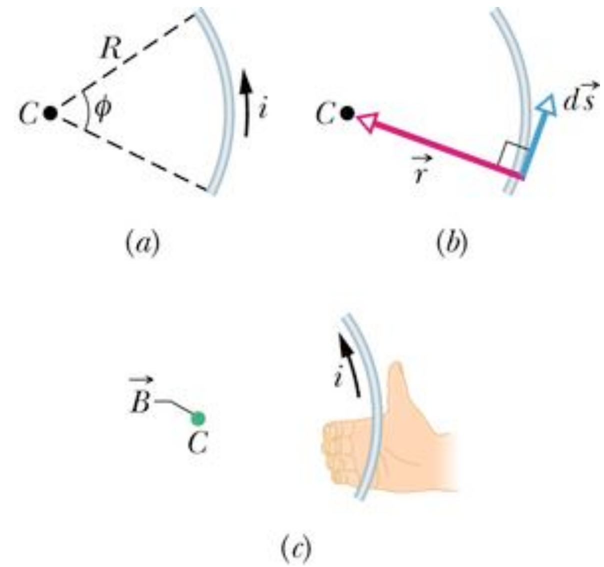
$$B = \frac{\mu_0 i}{2\pi R}$$

$$= \frac{(4\pi x 10^{-7}\, T.m / A)(500\, A)}{2\pi (100 m)}$$

$= 1\ \mu T!!$

Recall that the earth's magnetic field is $\sim 10^{-4} T = 100\ \mu T$

Biot-Savart Law



- A circular arc of wire of radius R carries a current *i*
- What is the magnetic field *at the center of the loop*?

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\overrightarrow{ds} \times \vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} \frac{idsR}{R^3} = \frac{\mu_0}{4\pi} \frac{iRd\phi}{R^2}$$

$$B = \frac{\mu_0}{4\pi} \int \frac{id\phi}{R} = \frac{\mu_0 i \Phi}{4\pi R}$$

Direction of B?? Not ***another*** right hand rule?!
TWO right hand rules!:
If your thumb points along the CURRENT, your fingers will point in the same direction as the FIELD.
If you curl our fingers around direction of CURRENT, your thumb points along FIELD!

Summary

- Magnetic fields from currents from **Biot-Savart's law**:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3}$$

- Two **right-hand rules** for direction of magnetic field
- **Straight currents** produce circular magnetic fields:
  $B=\mu_0 i/2\pi r$
- Current through **circular arc** produce magnetic field at center:
  $B = \mu_0 i \Phi/4\pi r$