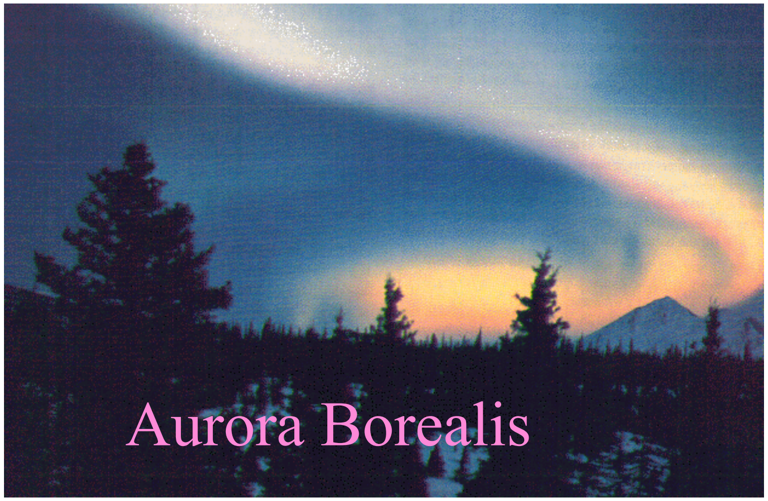
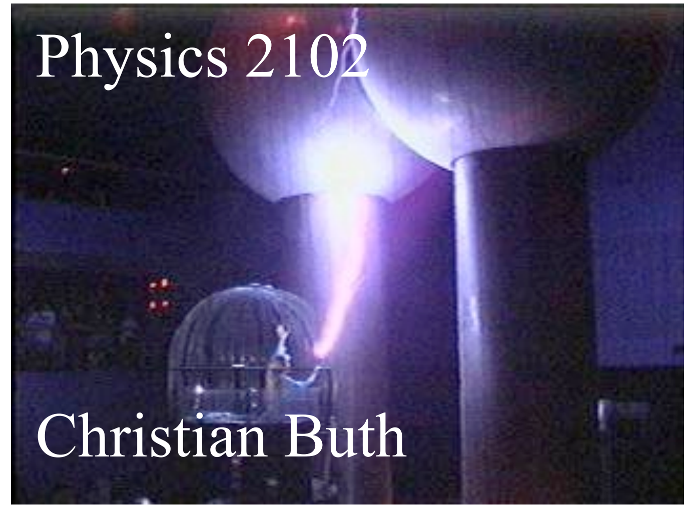




Physics 2102
Lecture 21
Magnetic fields 4

Version: 03/06/2009

Review

- **Hall effect** reveals that charge carriers in metals are negative (electrons)

- Charged particle with velocity perpendicular to magnetic field on **circular path**

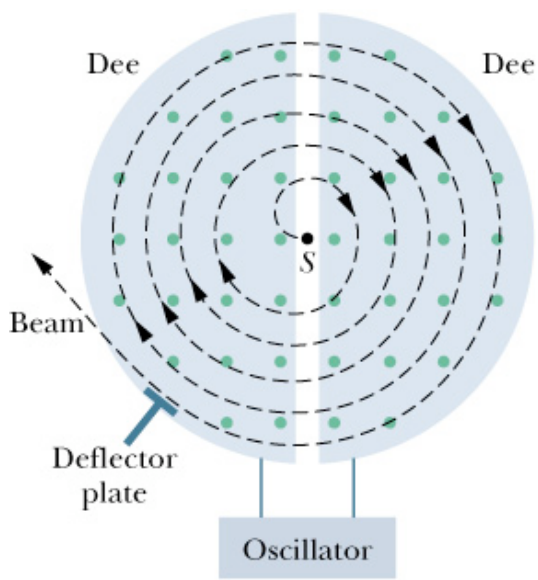
$$r = \frac{mv_\perp}{|q|B} \qquad T = \frac{2\pi m}{|q|B}$$

- Generally linear and circular motion superimposed: a **helix** with radius r and **pitch** $p = Tv_\parallel$

- Charged particles trapped in **magnetic bottle**; natural phenomenon: **aurora borealis**

- **Particle accelerators** create beams of charged particles

Cyclotron



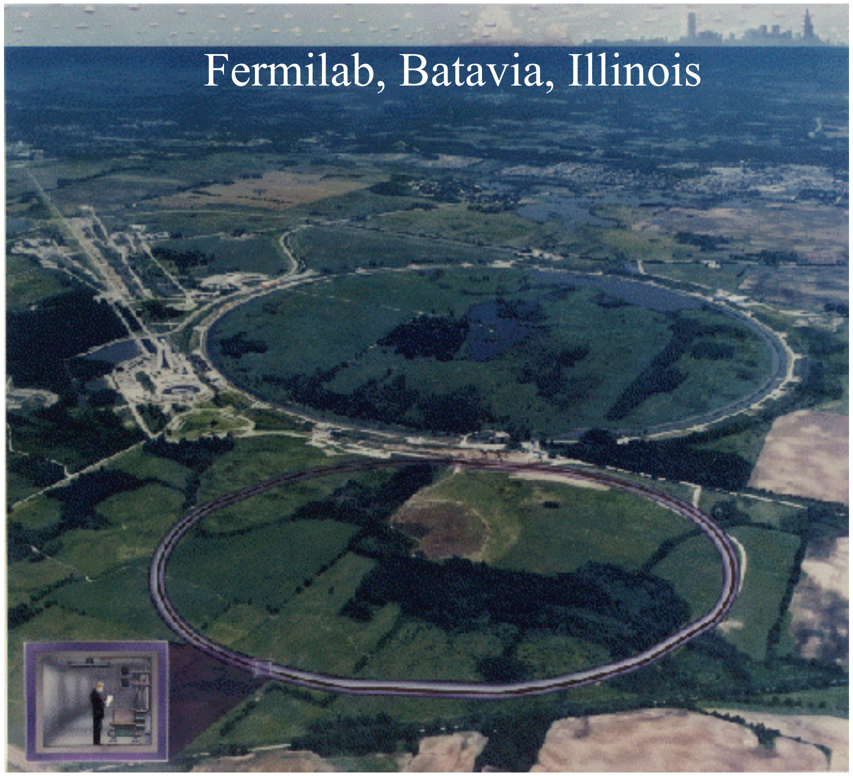
$$r = \frac{mv}{eB}$$

$$f = \frac{eB}{2\pi m}$$

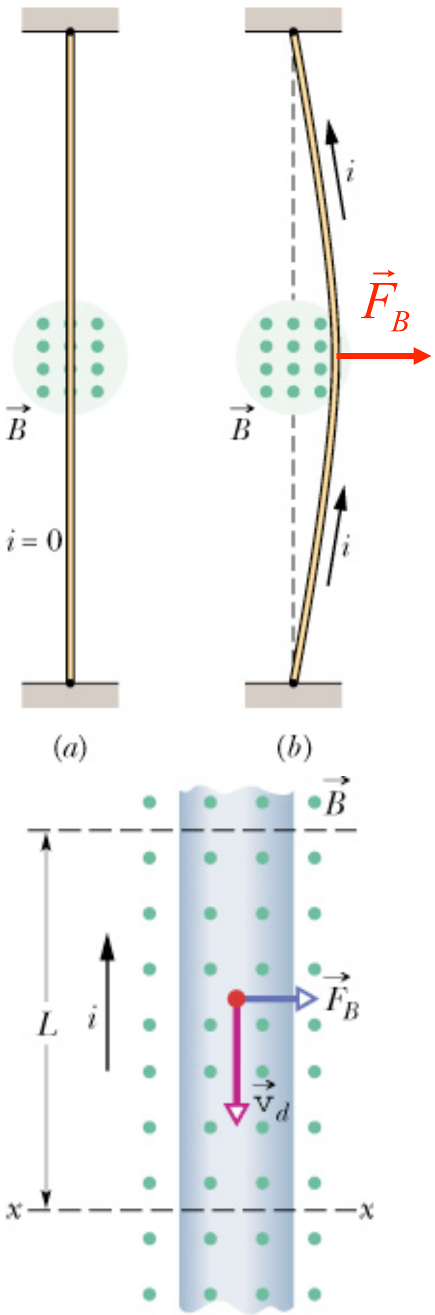
- Two hollow **D-shaped** objects (dees)
- Magnetic field forces particle on **spiral motion** (radius increases with particle speed)
- **Oscillating electric potential** changes charge of dees after each passing of particle in other dee
- **Resonance** condition $f = f_{osc}$
- **Relativistic effects** limit the particle speed that can be reached (frequency no longer independent of particle speed)

Synchrotron

- **Circular** (not spiral!) motion in a subterranean tunnel
- Magnetic field and frequency **vary** with particle speed
- **Fermi National Accelerator Laboratory** (Fermilab) has circumference of 6.3 km
- Fermilab can produce protons up to **1 TeV**

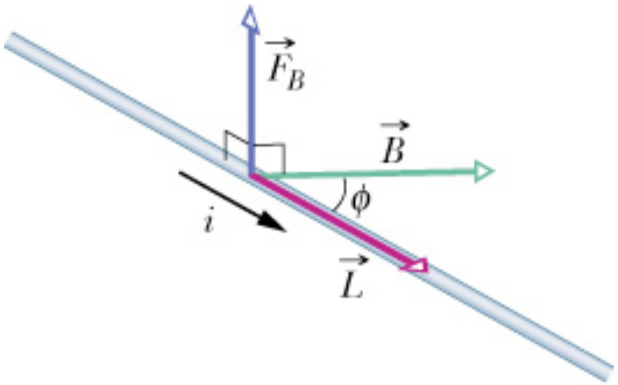


Magnetic Force on a Current-Carrying Wire

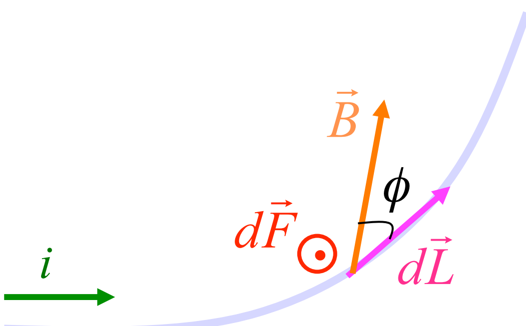


- Wire in uniform **magnetic field**
- **Experimentally**, we find that force is **perpendicular** on wire
- Force due to vector sum of **magnetic forces** on the moving electrons
- **Total charge** that flows through wire in time t is $q = i\,t = i\,L\,/\,v_d$
- **Magnetic force on wire** is $F_B = q\,v_d\,B\,\sin 90^0 = i\,L\,B$

General Force on Wire

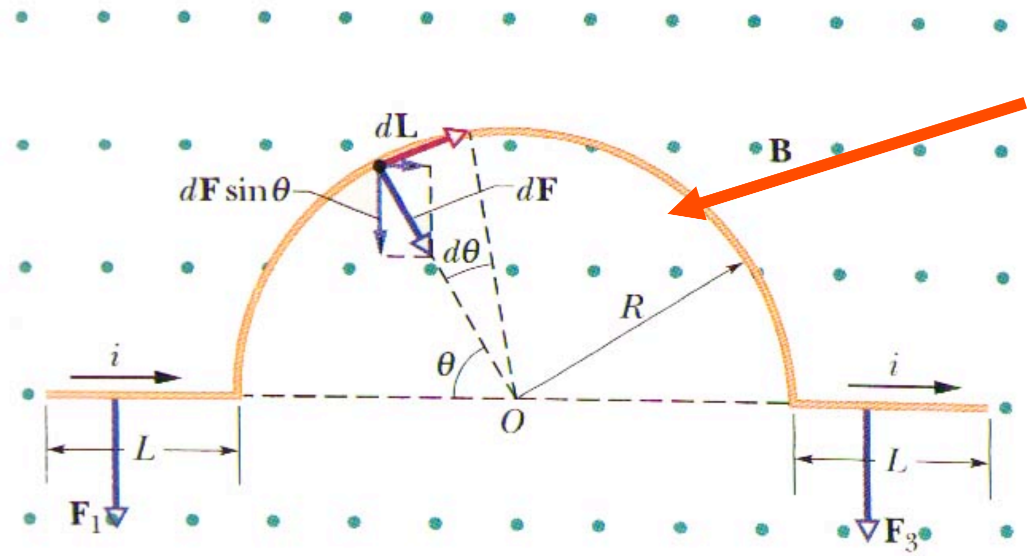


- Magnetic field angle ϕ with **straight wire**
- Magnetic force: $\boxed{\vec{F}_B = i\vec{L} \times \vec{B}}$
- Here ***L*** is vector with length L and and **direction of current**



- For wire with **arbitrary shape** decompose in elements dL
- Force on **element**: $\boxed{d\vec{F}_B = id\vec{L} \times \vec{B}}$
- **Net force**: $\boxed{\vec{F}_B = i\int d\vec{L} \times \vec{B}}$

Example



Wire with current *i*.
Magnetic field out of page.
What is net force on wire?

$$F_1 = F_3 = iLB$$

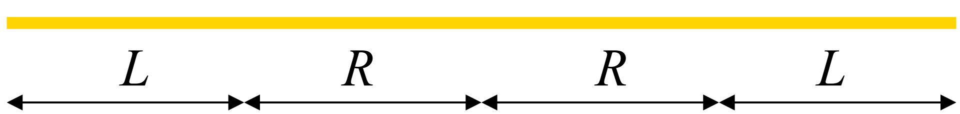
$$dF = iBdL = iBRd\theta$$

By symmetry, F_2 will only have a vertical component,

$$F_2 = \int_0^{\pi} \sin(\theta)dF = iBR\int_0^{\pi} \sin(\theta)d\theta = 2iBR$$

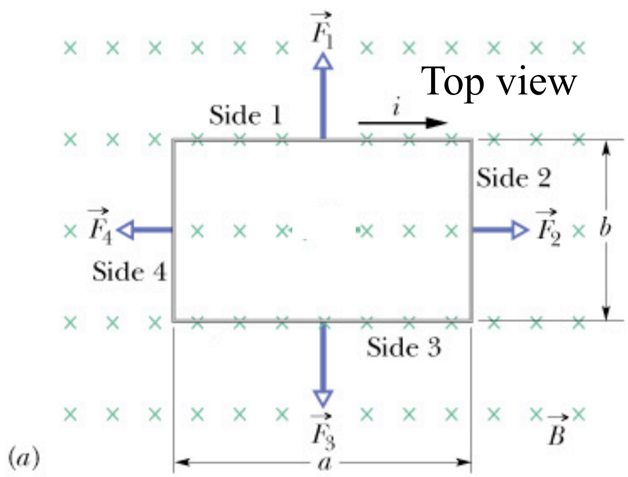
$$F_{\text{total}} = F_1 + F_2 + F_3 = iLB + 2iRB + iLB = 2iB(L+R)$$

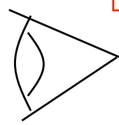
Notice that the force is the same as that for a straight wire,



and this would be true no matter what the shape of the central segment!

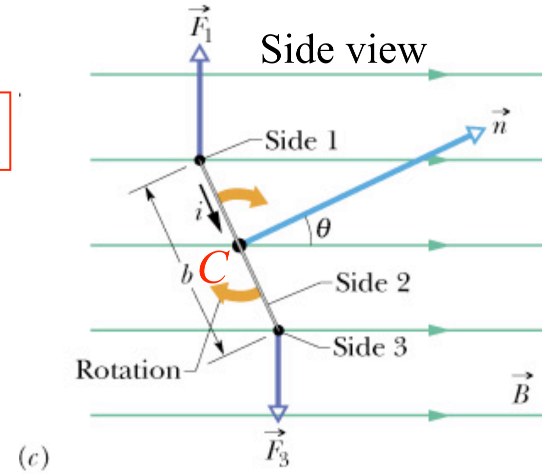
Magnetic Torque on Current Loop





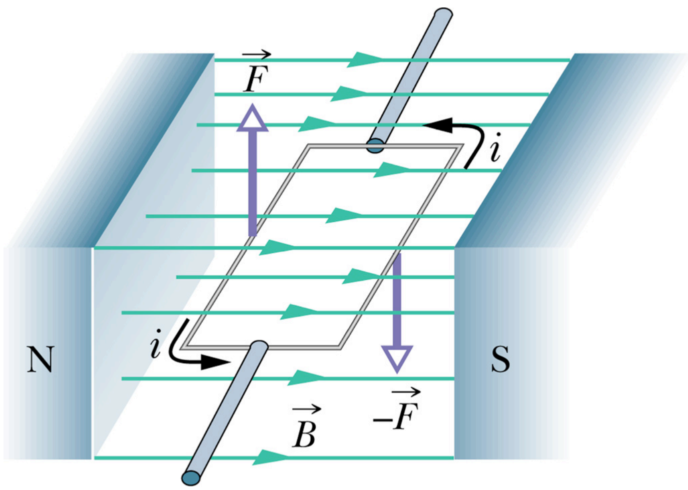
$\tau_{net} = iAB\sin\theta$

$F_{net} = 0$



- Same line of action (through center), equal magnitude, opposite direction, **zero effect**: $F_2 = F_4 = iaB\sin(90^0 - \theta) = iaB\cos\theta$
- Different line of action, equal magnitude, opposite direction, **rotates loop clockwise**: $F_1 = F_3 = iaB\sin 90^0 = iaB$
- Moment arm: $(b/2)\sin\theta$
- **Torque**: $\tau = \tau_1 + \tau_3 = (iabB/2)\sin\theta + (iabB/2)\sin\theta = iabB\sin\theta$

Electric Motors



- For a coil with ***N*** turns, torque **τ** = ***N*** *I A B* sin*θ*, where *A* is the area of coil
- Reverse current in coil with **commuter** after every half turn reverses the forces
- Coil turns: an **electric motor**

Magnetic Dipole Moment

We just showed: $\tau = NiAB\sin\theta$

N: number of turns in coil

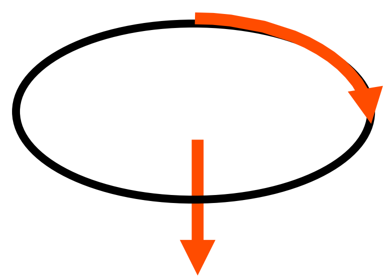
A: area of coil

Magnetic dipole moment μ

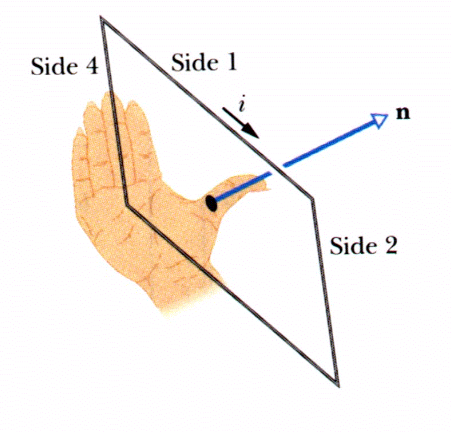
Right hand rule:
curl fingers in direction of current;
thumb points along μ

$$\vec{\mu} = (NiA)\hat{n}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



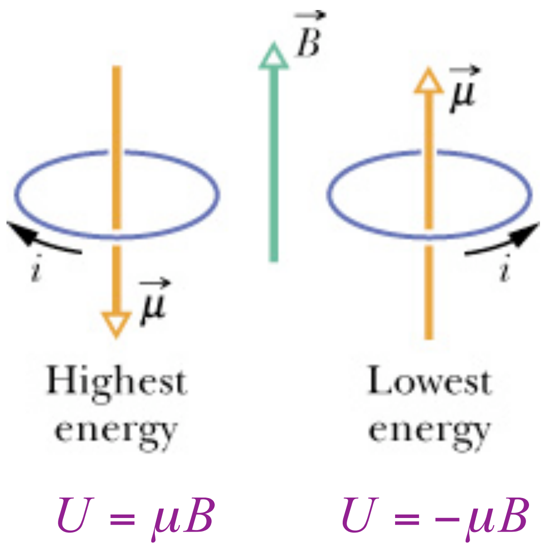
$\vec{\mu}, \hat{n}$



(b)

As in the case of electric dipoles, magnetic dipoles tend to align with the magnetic field

Magnetic Potential Energy

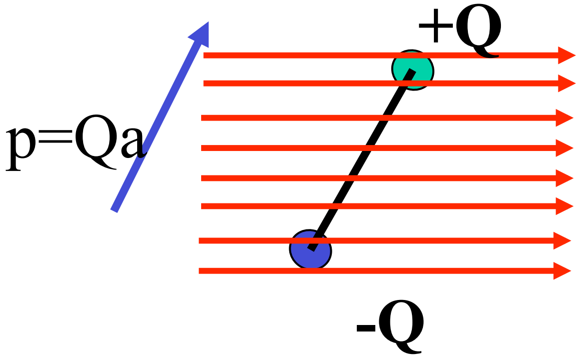


- Torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$
- In analogy to electric dipole in homogenous electric field we define **magnetic potential energy**

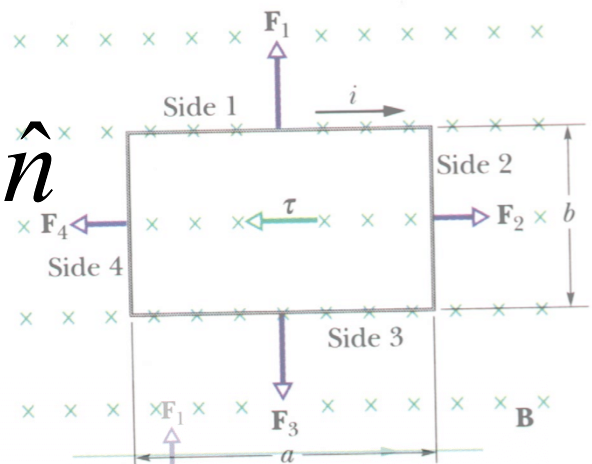
$$U = -\vec{\mu} \cdot \vec{B}$$

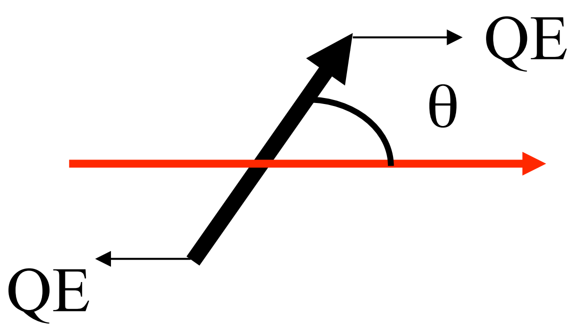
- For both positions in figure **torque zero** (minimum and maximum)
- **External agent** does work on dipole: $W_a = U_f - U_i$

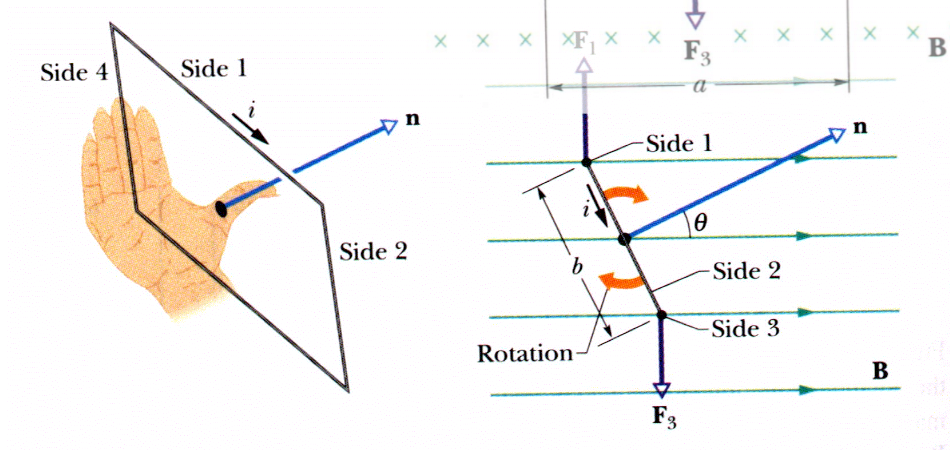
Electric versus Magnetic Dipoles



$$\vec{\mu} = (NiA)\hat{n}$$





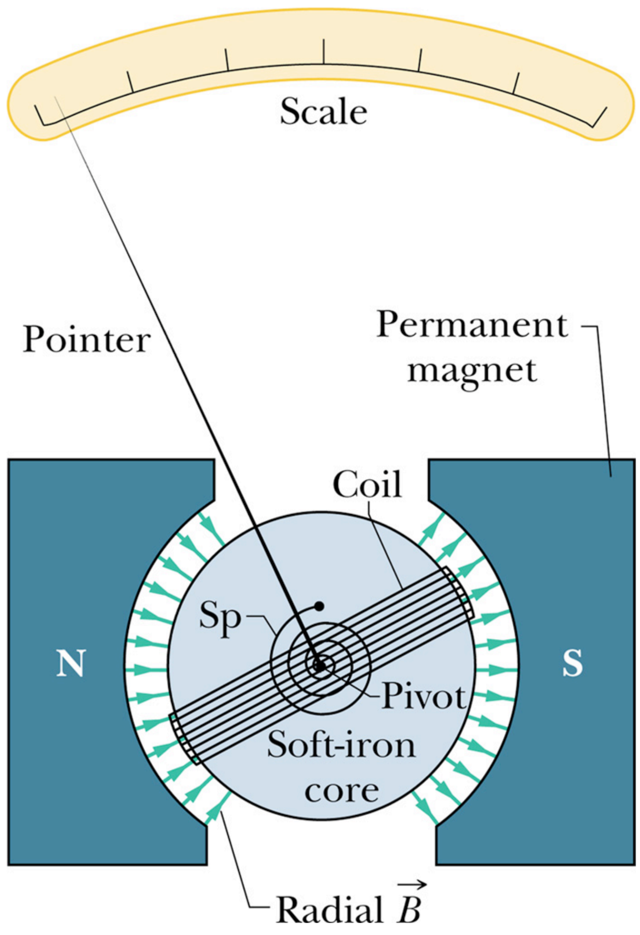


(b) (c)

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Galvanometer



- **Analog** ammeters and voltmeters contain **galvanometer**
- Current through coil generates **torque**
- **Radial** magnetic field
- Always field **perpendicular** to normal vector of coil
- Spring provides **countertorque**:

$$\tau = -\kappa\phi = NiAB$$

Summary

- **Cyclotrons** and **synchrotrons** to accelerate particles
- Wires carrying currents produce forces on each other
- Parallel currents **attract**, antiparallel currents **repel**
- For a straight wire: $\vec{F}_B = i\vec{L} \times \vec{B}$, generally: $\vec{F}_B = i\int d\vec{L} \times \vec{B}$
- **Current loop** is **magnetic dipole;** in uniform magnetic field

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad \vec{\mu} = (NiA)\hat{n} \qquad U = -\vec{\mu} \cdot \vec{B}$$

- **Right-hand rule** gives direction of moment
- **Magnetic potential energy** of magnetic dipole in uniform magnetic field