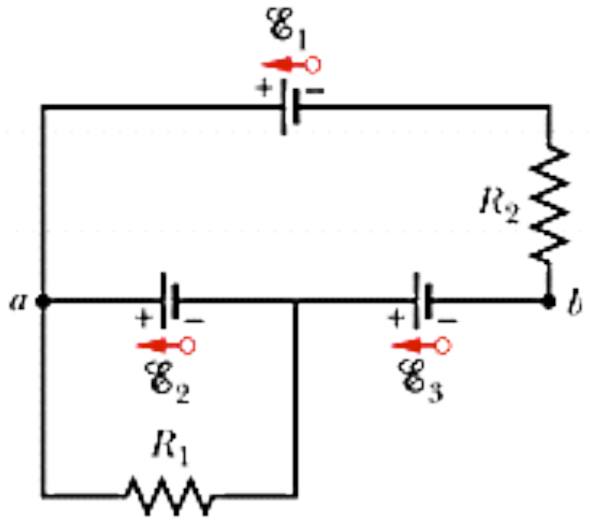
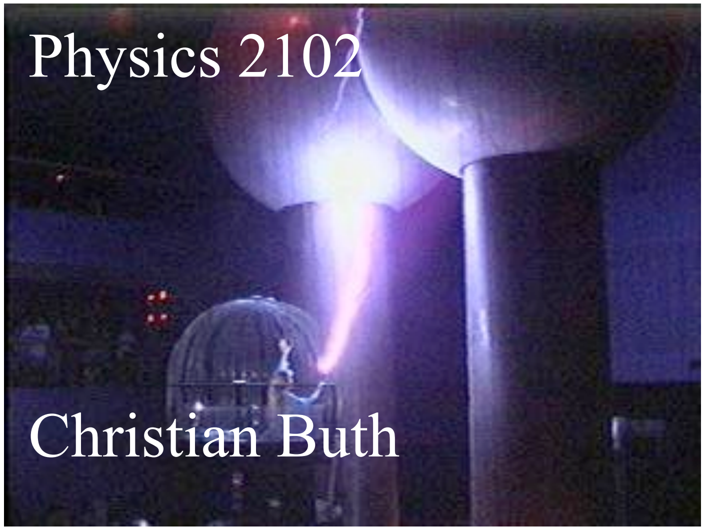
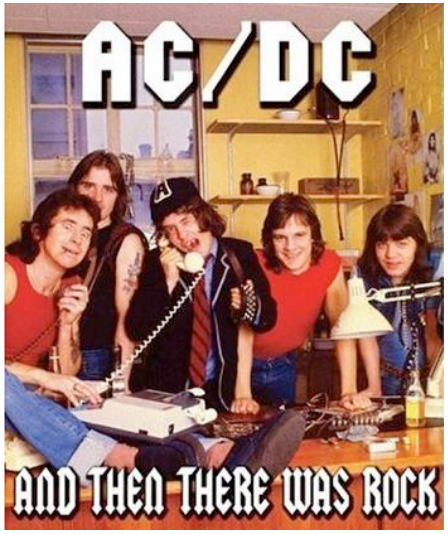




Physics 2102
Lecture 17
DC circuits, RC circuits



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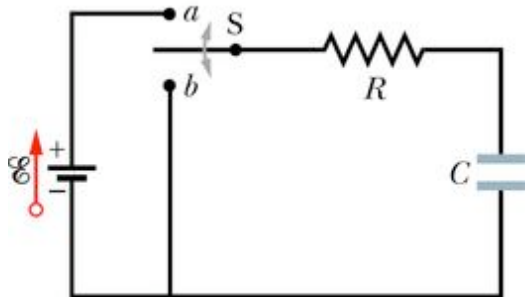


Review

- The potential of ideal emf devices does not depend on the current; real emf devices have **internal resistance**
- **Kirchhoff's junction rule:**

KJR : The sum of the currents entering any junction is equal to the sum of the currents leaving the junction.

- **Resistors in parallel** replace by equivalent resistance

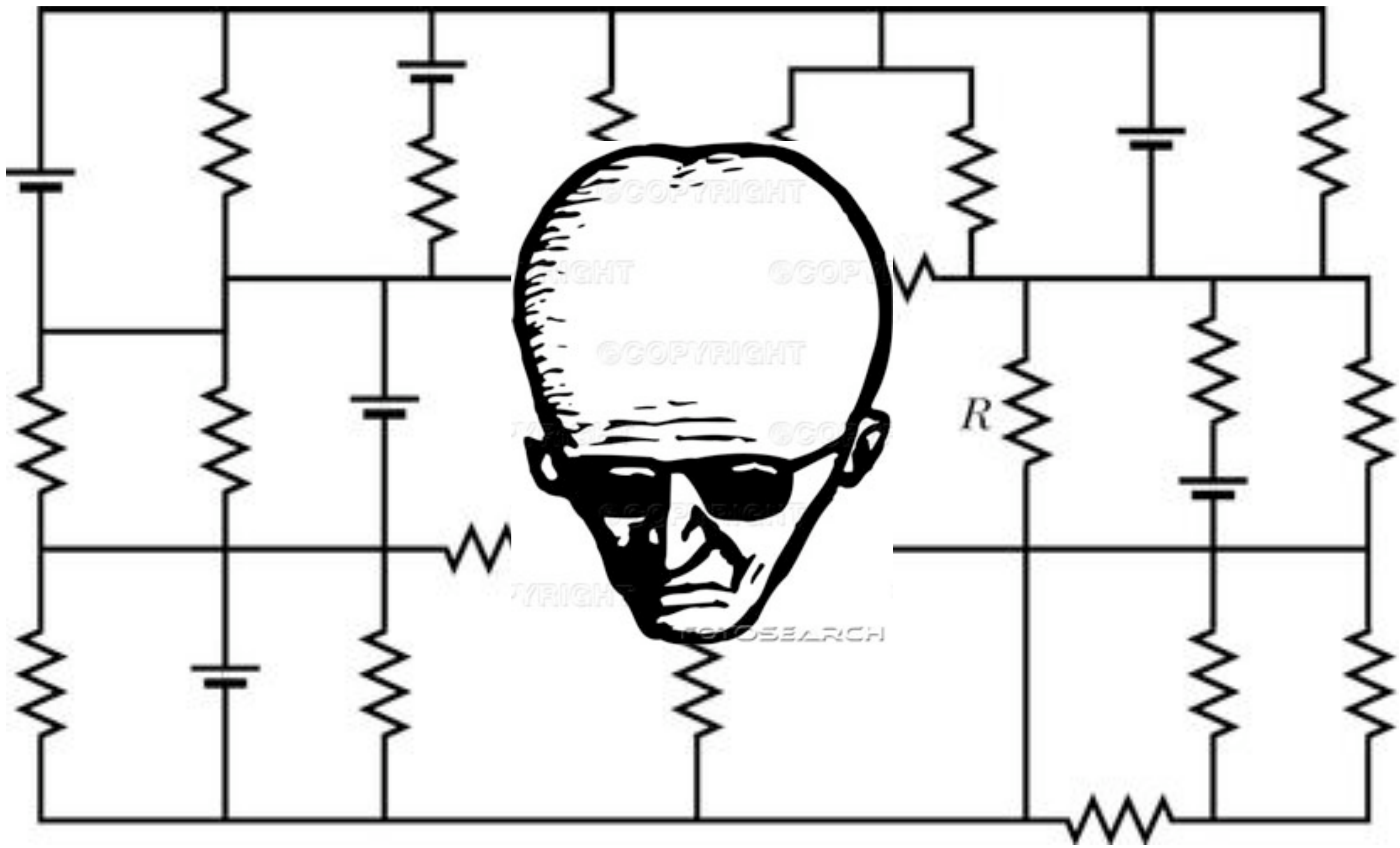
$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$$

- **Resistors in series** replace by equivalent resistance

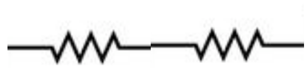
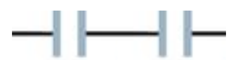
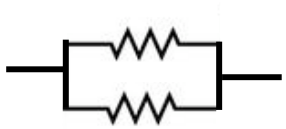
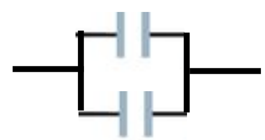
$$R_{eq} = \sum_{j=1}^{n} R_j$$

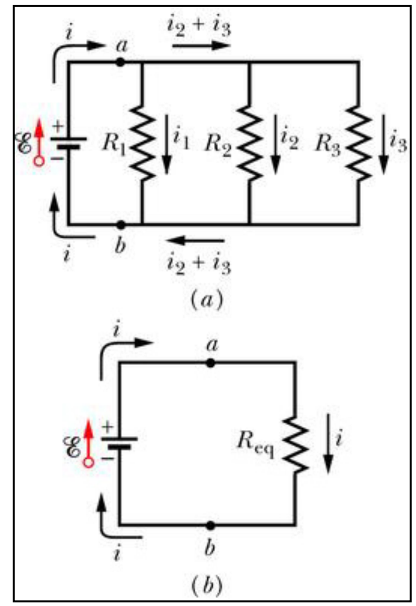
- **Ammeter** measures current; **voltmeter** measures voltage

How to Solve Multi-Loop Circuits



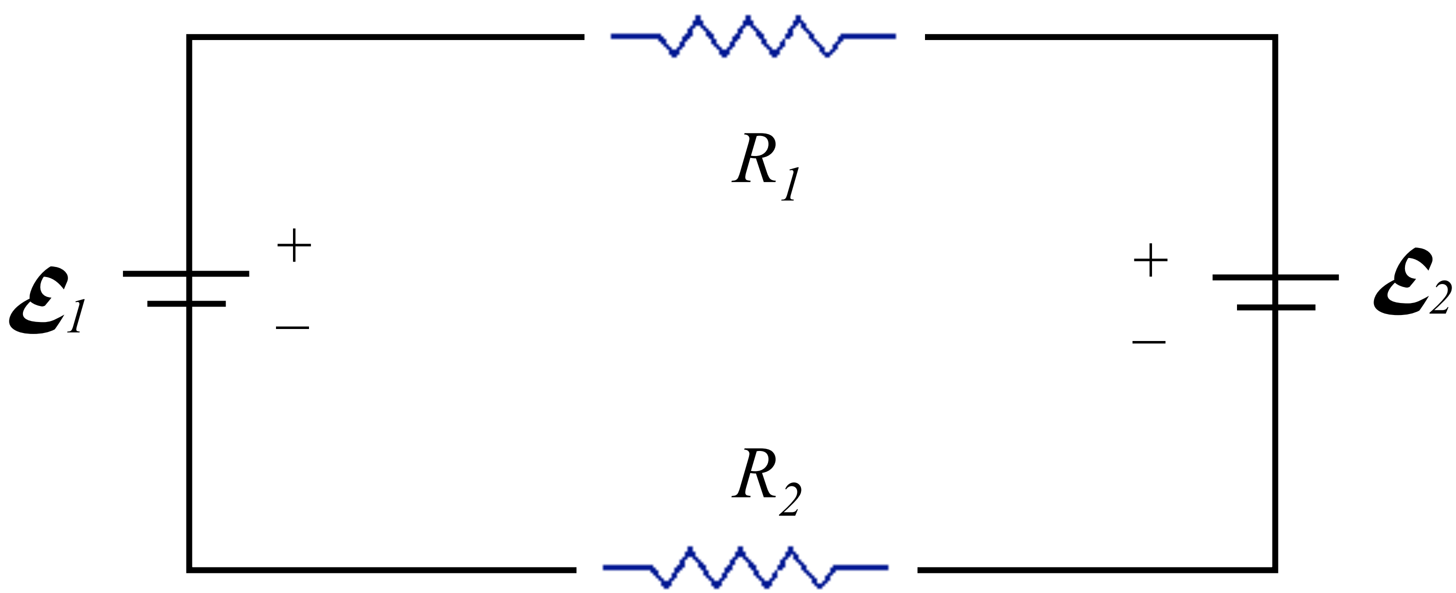
Step I: Simplify "Compile" Circuits

	Resistors	Capacitors
		
Key formula:	$V=iR$	$Q=CV$
In series:	same current	same charge
	$R_{eq}=\sum R_j$	$1/C_{eq}=\sum 1/C_j$
		
In parallel:	same voltage	same voltage
	$1/R_{eq}=\sum 1/R_j$	$C_{eq}=\sum C_j$
		



Step II: Apply Loop Rule

Around *every* loop add $+\mathcal{E}$ if you cross a battery from minus to plus, $-\mathcal{E}$ if plus to minus, and $-iR$ for each resistor. Then sum to Zero: $+\mathcal{E}_1 - \mathcal{E}_2 - iR_1 - iR_2 = 0$.

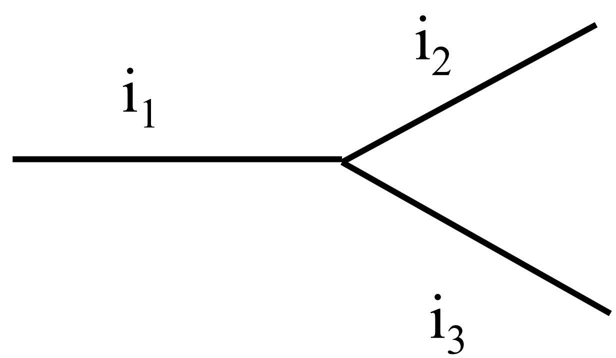


Conservation of ENERGY!

Step II: Apply Junction Rule

At every junction sum the ingoing currents and outgoing currents and set them equal.

$$i_1 = i_2 + i_3$$



Conservation of CHARGE!

Step III: Equations to Unknowns

Continue Steps I–III until you have as many equations as unknowns!

Given: $\mathcal{E}_1$, $\mathcal{E}_2$, i, R_1, R_2

$$+\mathcal{E}_1 - \mathcal{E}_2 - i_1 R_1 - i_2 R_2 = 0$$

and

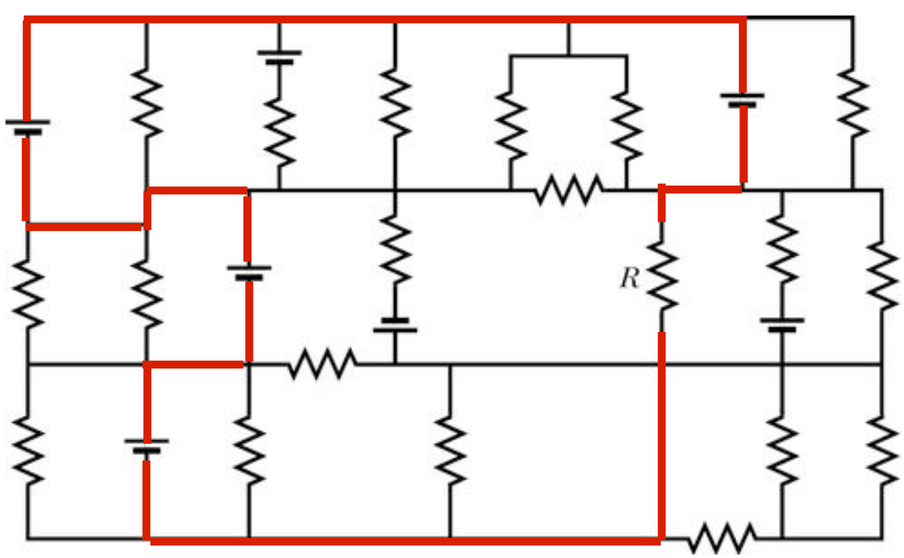
$$i = i_1 + i_2$$

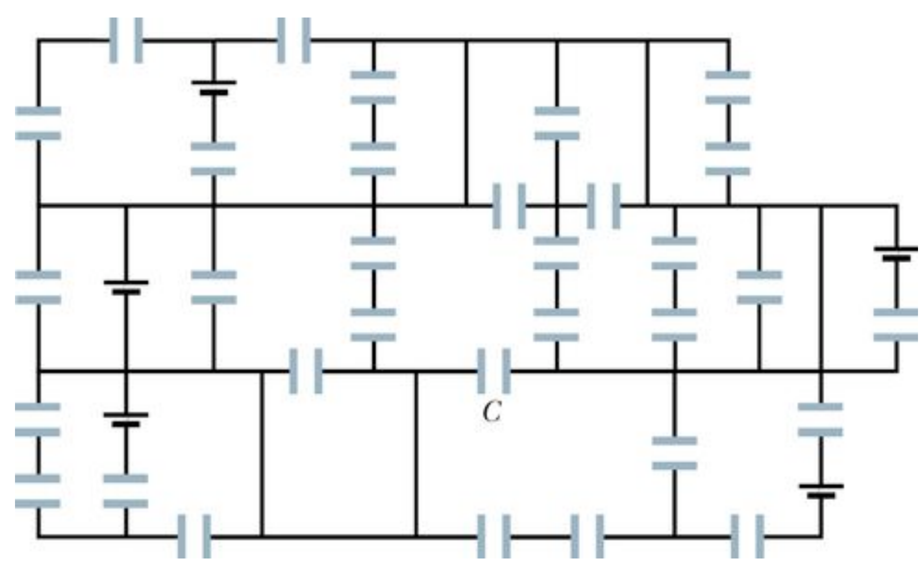
Solve for i_1, i_2

Monster Mazes

If all resistors have a resistance of 4Ω, and all batteries are ideal and have an emf of 4V, what is the current through R?

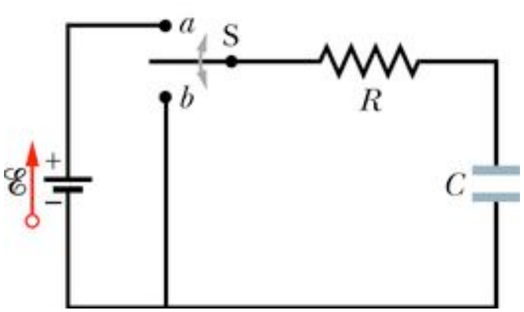
If all capacitors have a capacitance of 6μF, and all batteries are ideal and have an emf of 10V, what is the charge on capacitor C?





RC Circuits: Charging a Capacitor

In these circuits, current will change for a while, and then stay constant.
We want to solve for current as a function of time $i(t)$.
The charge on the capacitor will also be a function of time: $q(t)$.
The voltage across the resistor and the capacitor also change with time.
To charge the capacitor, close the switch on *a*.

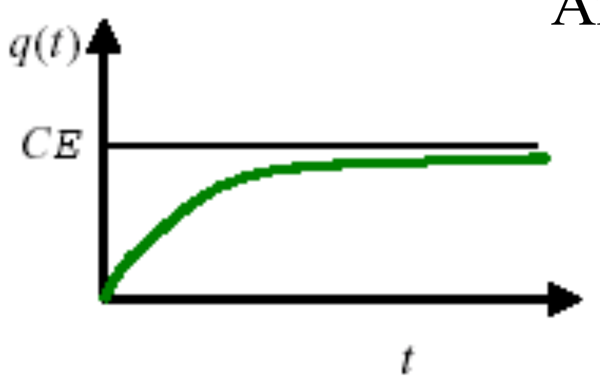


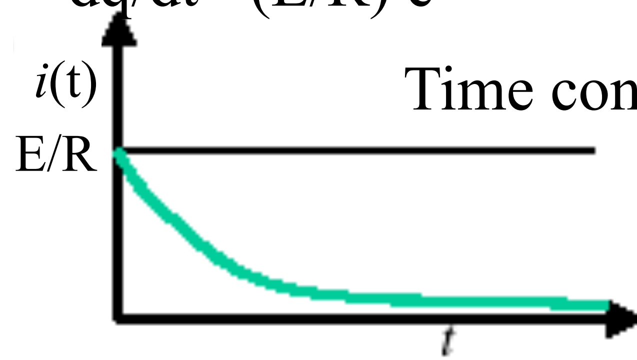
$$\mathrm{E} + V_R(t) + V_C(t) = 0$$
$$\mathrm{E} - i(t)R - q(t)/C = 0$$
$$\mathrm{E} - (dq(t)/dt)\, R - q(t)/C = 0$$

A differential equation for q(t)! The solution is:

$$q(t) = C\mathrm{E}(1 - e^{-t/RC})$$

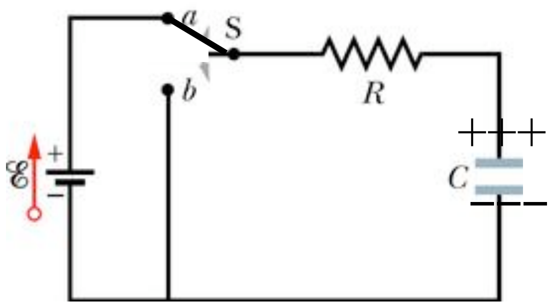
And then $i(t) = dq/dt = (\mathrm{E}/R)\, e^{-t/RC}$





Time constant=RC

RC Circuits: Discharging a Capacitor

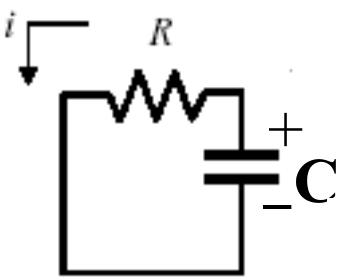


Assume the switch has been closed on a for a long time: the capacitor will be charged with Q=CE.

Then, close the switch on *b*: charges find their way across the circuit, establishing a current.

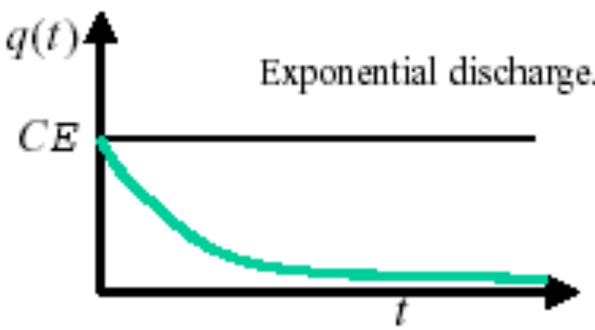
$V_R+V_C=0$

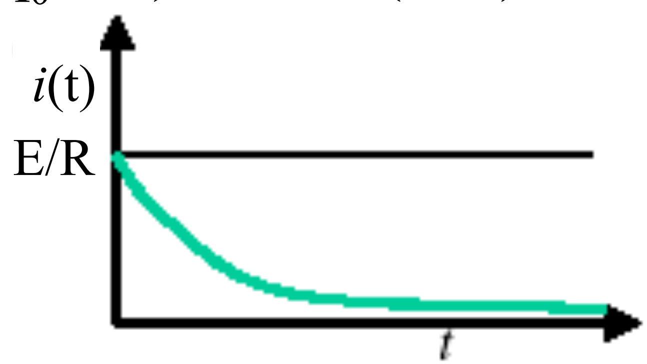
$-i(t)R-q(t)/C=0 \Rightarrow (dq/dt)R+q(t)/C=0$



Solution: $q(t)=q_0e^{-t/RC}=CEe^{-t/RC}$

$i(t) = dq/dt = (q_0/RC)\ e^{-t/RC} = (E/R)\ e^{-t/RC}$





Summary

- Technique to **solve multiloop circuits**
    1. Simplify "compile" circuits
    2. Apply loop rule
    3. Equations to unknowns

- **RC circuits**: simple circuit with time-varying current; time constant is $\tau = RC$

- **Charging** a capacitor: $q(t) = CE(1-e^{-t/RC})$

- **Discharging** a capacitor: $q(t)=CEe^{-t/RC}$