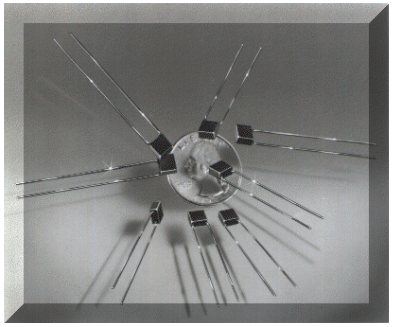
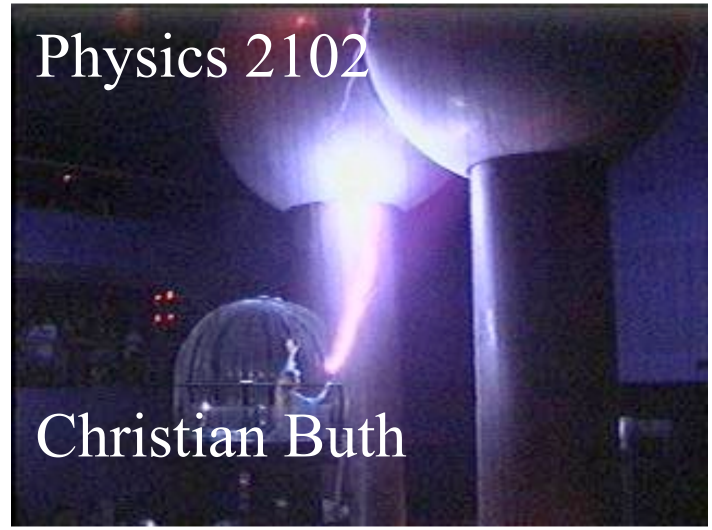
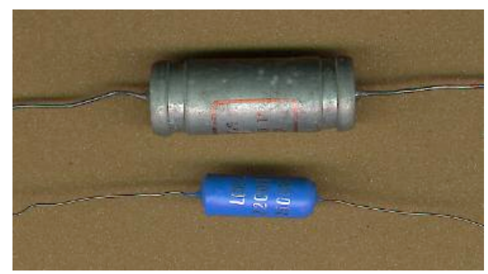




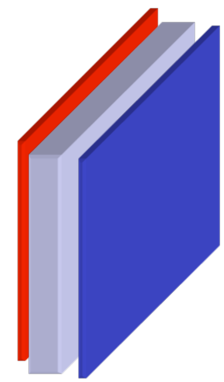
Physics 2102
Lecture 12
Capacitors 2

Version: 02/09/2009



Review

- Any two charged conductors form a **capacitor**

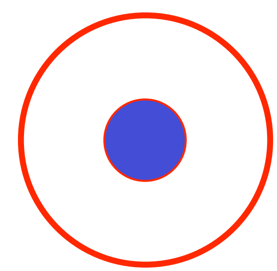


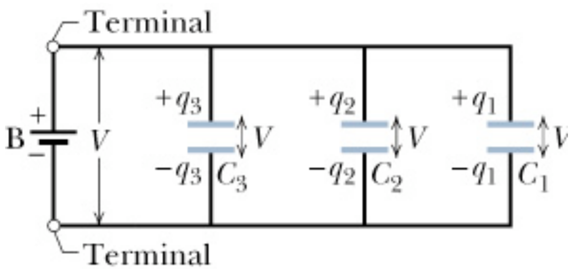
- **Capacitance**: $C = Q/V$

- Simple Capacitors:

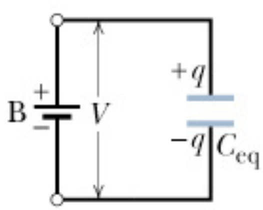
Parallel plates: $C = \varepsilon_0 A/d$

Spherical: $C = 4\pi \varepsilon_0 ab/(b-a)$





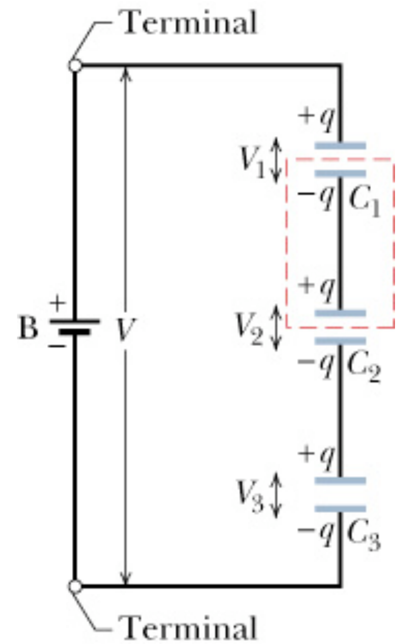
(a)



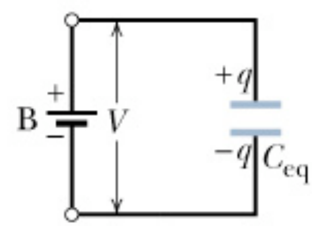
(b)

Equivalent Capacitor

Consider the combination of capacitors shown in the figure to the left and to the right (upper part). We will substitute these combinations of capacitor with a single capacitor C_{eq} that is "electrically equivalent" to the capacitor group it substitutes.



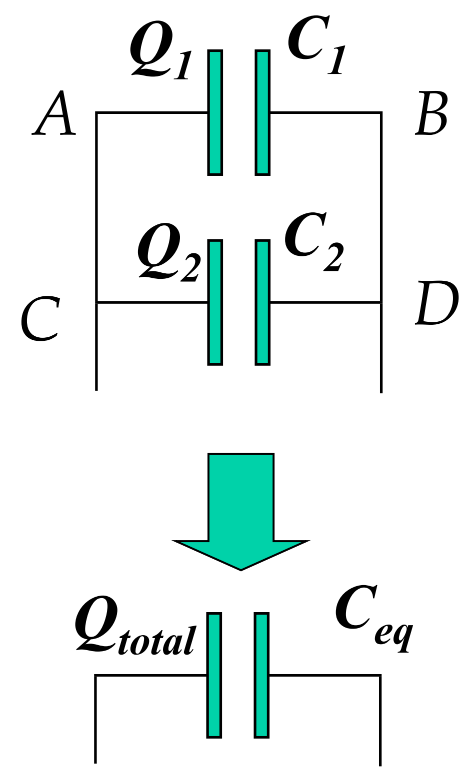
(a)



(b)

Capacitors in Parallel

- A **wire** is an equipotential surface!
- Capacitors in parallel have **same** potential difference but **not always** same charge!
- $V_{AB} = V_{CD} = V$
- $Q_{total} = Q_1 + Q_2$
- $C_{eq}V = C_1V + C_2V$
- $C_{eq} = C_1 + C_2$
- Equivalent **parallel** capacitance = sum of capacitances

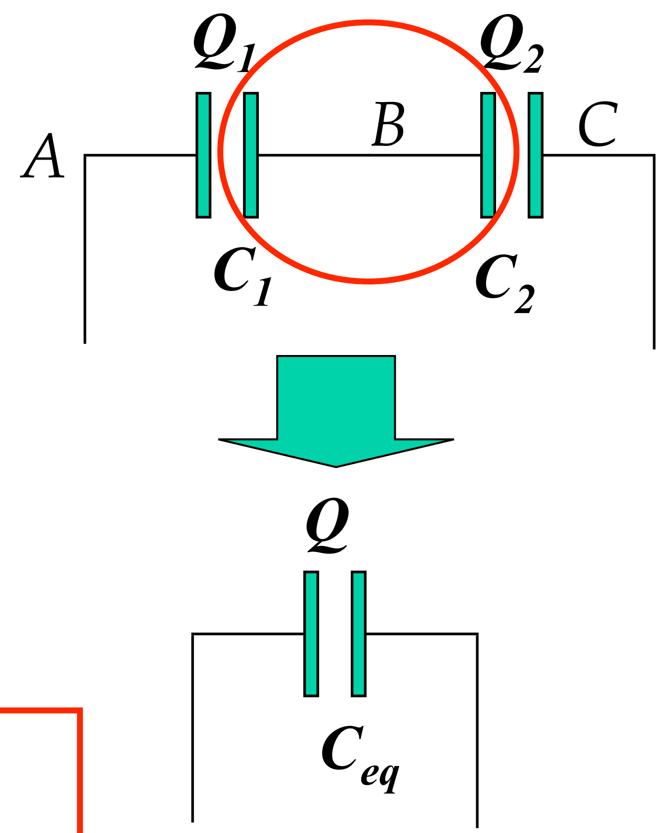


Capacitors in Series

- $Q_1 = Q_2 = Q$ (WHY??)
- $V_{AC} = V_{AB} + V_{BC}$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



SERIES:
- *Q* is **same** for all capacitors
- Total potential difference = sum of *V*

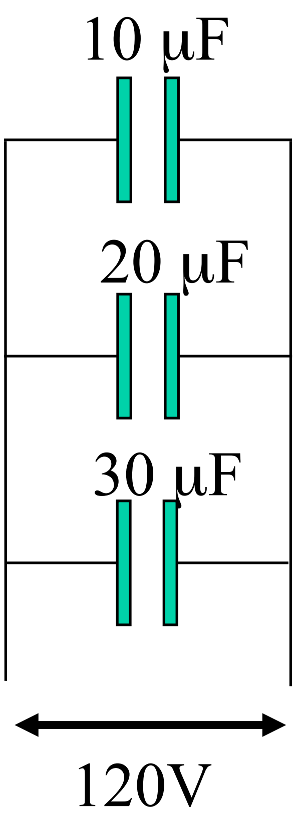
Example 1

What is the charge on each capacitor?

- $Q = CV$; $V = 120$ V
- $Q_1 = (10\ \mu F)(120V) = 1200\ \mu C$
- $Q_2 = (20\ \mu F)(120V) = 2400\ \mu C$
- $Q_3 = (30\ \mu F)(120V) = 3600\ \mu C$

Note that:

- Total charge (7200 μC) is shared between the 3 capacitors in the ratio $C_1{:}C_2{:}C_3$ — i.e. 1:2:3

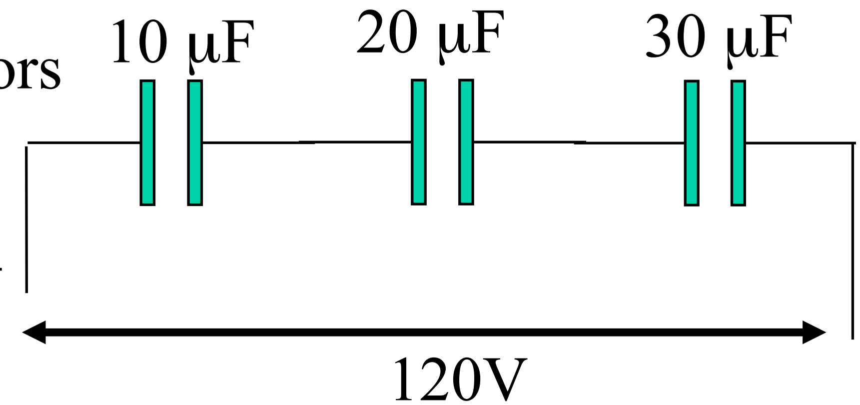


Example 2

What is the potential difference across each capacitor?

- $Q = CV$; Q is same for all capacitors
- Combined C is given by:

$$\frac{1}{C_{eq}} = \frac{1}{(10\mu F)} + \frac{1}{(20\mu F)} + \frac{1}{(30\mu F)}$$

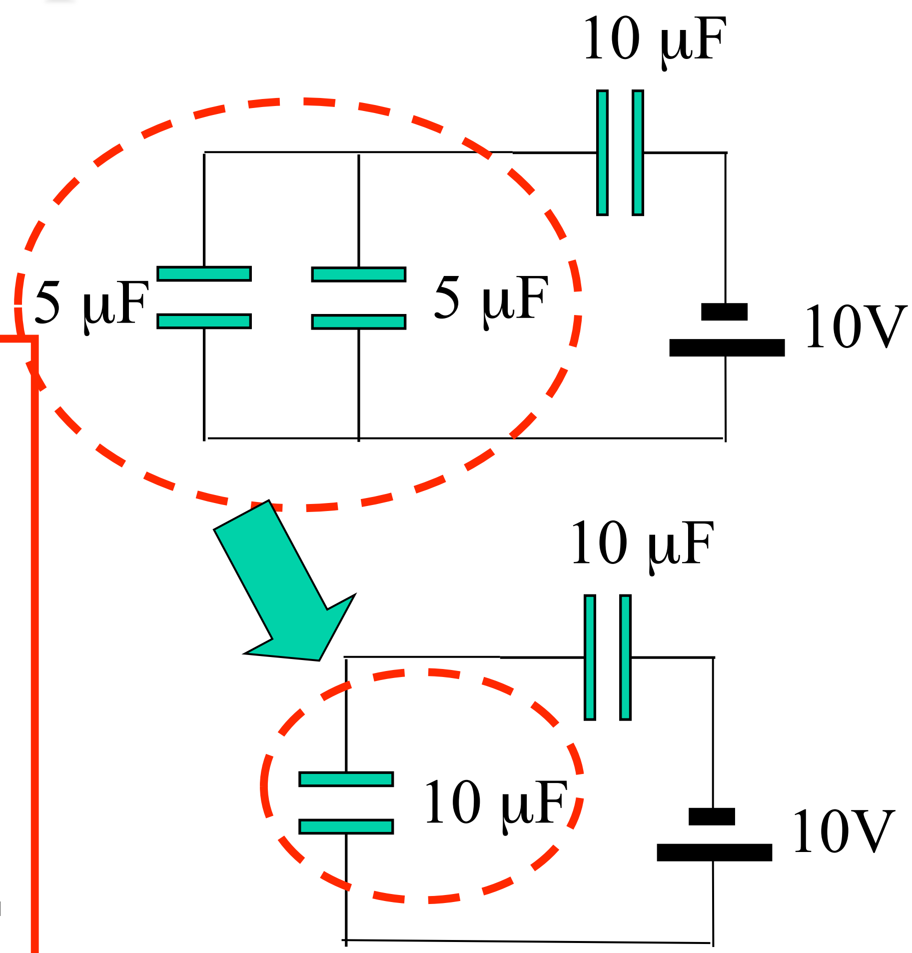


- $C_{eq} = 5.46\ \mu F$
- $Q = CV = (5.46\ \mu F)(120V) = 655\ \mu C$
- $V_1 = Q/C_1 = (655\ \mu C)/(10\ \mu F) = 65.5\ V$
- $V_2 = Q/C_2 = (655\ \mu C)/(20\ \mu F) = 32.75\ V$
- $V_3 = Q/C_3 = (655\ \mu C)/(30\ \mu F) = 21.8\ V$

Note: 120V is shared in the ratio of **inverse** capacitances i.e. 1:(1/2):(1/3) (largest C gets smallest V)

Example 3

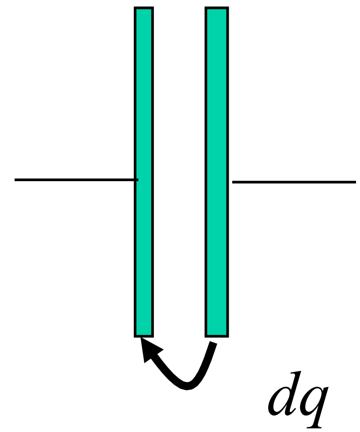
In the circuit shown, what is the charge on the 10μF capacitor?



- The two **5μF** capacitors are in parallel
- Replace by **10μF**
- Then, we have two **10μF** capacitors in series
- So, there is 5V across the **10μF** capacitor of interest
- Hence, Q = (**10μF**)(5V) = **50μC**

Energy Stored in a Capacitor

- Start out with **uncharged** capacitor
- Transfer **small amount** of charge dq from one plate to the other **until** charge on each plate has magnitude Q
- How much **work** is needed?



$$U = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{CV^2}{2}$$

Energy Stored in Electric Field

- Energy in capacitor: $U = Q^2/(2C) = CV^2/2$
- View the energy as stored in **electric field**
- For example, parallel plate capacitor: **energy density** = energy/volume = u

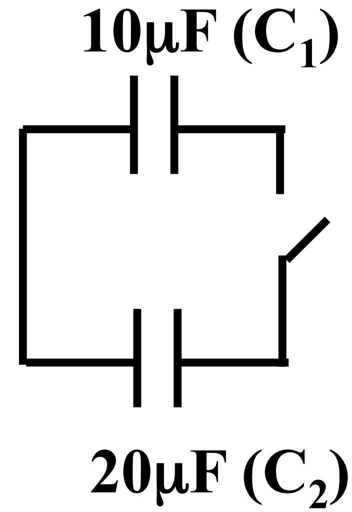
$$u = \frac{Q^2}{2CAd} = \frac{Q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)Ad} = \frac{Q^2}{2\varepsilon_0 A^2} = \frac{\varepsilon_0}{2}\left(\frac{Q}{\varepsilon_0 A}\right)^2 = \frac{\varepsilon_0 E^2}{2}$$

volume = Ad

General expression for any region with vacuum (or air)

Example

- **10μF capacitor is initially charged to 120V. 20μF capacitor is initially uncharged.**
- **Switch is closed, equilibrium is reached.**
- **How much energy is dissipated in the process?**



Initial charge on 10μF: (10μF)(120V)= 1200μC

After switch is closed, let charges = Q_1 and Q_2.

Charge is conserved: $Q_1 + Q_2 = 1200\mu C$

Also, V_{final} is same: $\dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} \Rightarrow Q_1 = \dfrac{Q_2}{2} \Rightarrow$

- $Q_1 = 400\mu C$
- $Q_2 = 800\mu C$
- $V_{final} = Q_1/C_1 = 40$ V

Initial energy stored = $(1/2)C_1 V_{initial}^2 = (0.5)(10\mu F)(120)^2 = 72mJ$

Final energy stored = $(1/2)(C_1 + C_2)V_{final}^2 = (0.5)(30\mu F)(40)^2 = 24mJ$

Energy lost (dissipated) = 48mJ

Summary

- **Capacitors in series**: same charge, not necessarily equal potential; equivalent capacitance $1/C_{eq}=1/C_1+1/C_2+\ldots$
- **Capacitors in parallel**: same potential; not necessarily same charge; equivalent capacitance $C_{eq}=C_1+C_2+\ldots$
- **Energy in a capacitor**: $U=Q^2/2C=CV^2/2$
- **Energy density**: $u=\varepsilon_0 E^2/2$