

Physics 2102

Lecture 11

Capacitors 1



Version: 02/06/2009



Review

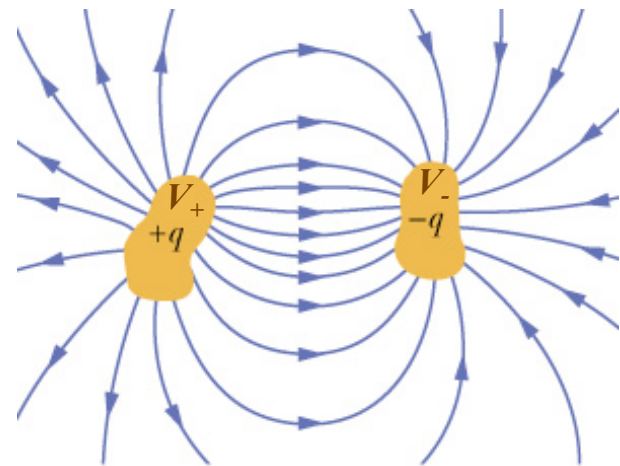
- **Electric potential energy**: work used to build the system, charge by charge:

$$U = \frac{1}{4\pi\epsilon_0} \sum_{\substack{i,j=1 \\ i < j}}^n \frac{q_i q_j}{r_{ij}}$$

- Charges move to make surface of **conductors** equipotentials
- Thus conductors **change the field** around them!
- Charge density and electric field are higher on **sharp points** of conductor

Capacitor

- **Use:** storing and releasing electric charge / energy
- **Capacitor:** any two conductors, one with charge $+Q$, other with charge $-Q$
- Potential difference between conductors: V



Electrolytic (1940-70)
Electrolytic (new)

Paper (1940-70)

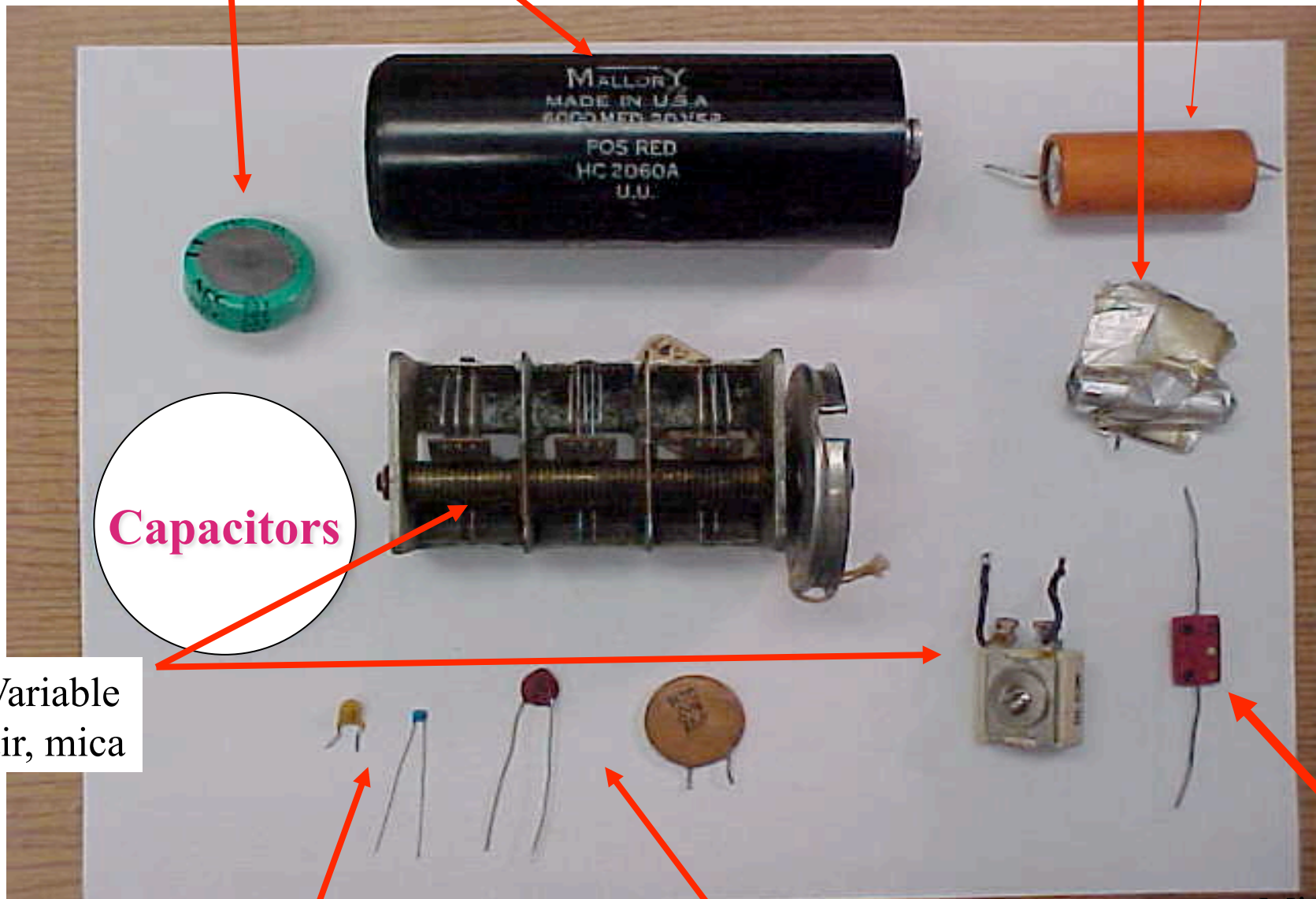
Capacitors

Variable
air, mica

Tantalum (1980 on)

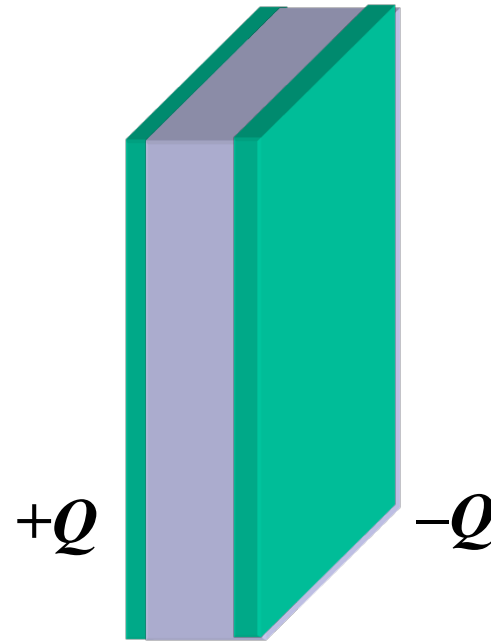
Ceramic (1930 on)

Mica
(1930-50)



Capacitance 1

- **Capacitance** quantifies the charge that can be stored in a capacitor
- Depends only on **geometrical** factors and the **material** between the two conductors
- e.g. Area of conductors, separation; space filled with air, plastic, etc.




(We first focus on capacitors where gap is filled by **air**!)

Capacitance 2

- Potential difference between conductors: V , capacitance C of capacitor, charge Q on capacitor:

$$Q = CV$$

- **Unit of capacitance:** **Farad** (F) = Coulomb/Volt
- **Most** electronic capacitors:
 - micro-Farad (μF) — 10^{-6} F
 - nano-Farad (pF) — 10^{-9} F
 - pico-Farad (pF) — 10^{-12} F
- **New technology:** compact 1 F capacitors
- **Symbol** in electric circuit: 

Parallel Plate Capacitor

We want *capacitance*: $C=Q/V$

E field between the plates: (Gauss' Law)

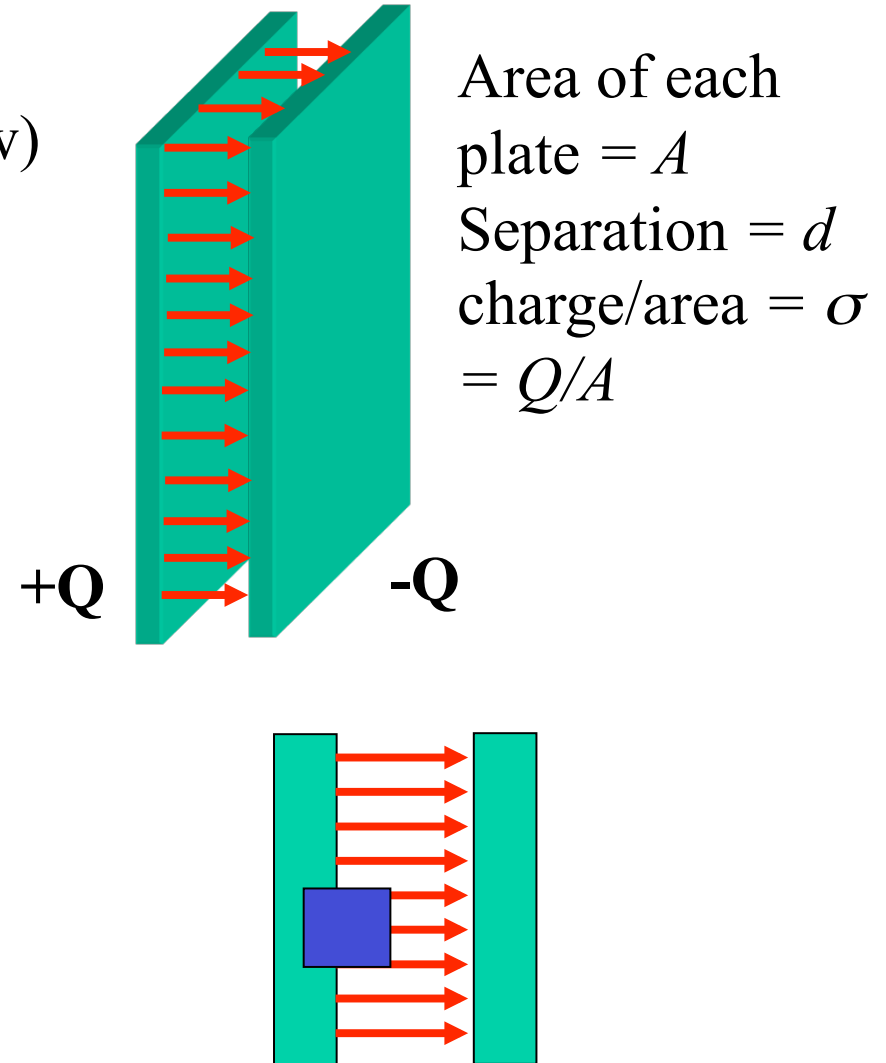
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Relate E to potential difference V :

$$V = \int_0^d \vec{E} \cdot d\vec{x} = \int_0^d \frac{Q}{\epsilon_0 A} dx = \frac{Qd}{\epsilon_0 A}$$

What is the capacitance C ?

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$



Parallel Plate Capacitor — Example

- A huge parallel plate capacitor consists of two square metal plates of side 50 cm, separated by an air gap of 1 mm
- What is the capacitance?

$$C = \epsilon_0 A/d$$

$$= (8.85 \times 10^{-12} \text{ F/m})(0.25 \text{ m}^2)/(0.001 \text{ m})$$

$$= 2.21 \times 10^{-9} \text{ F}$$

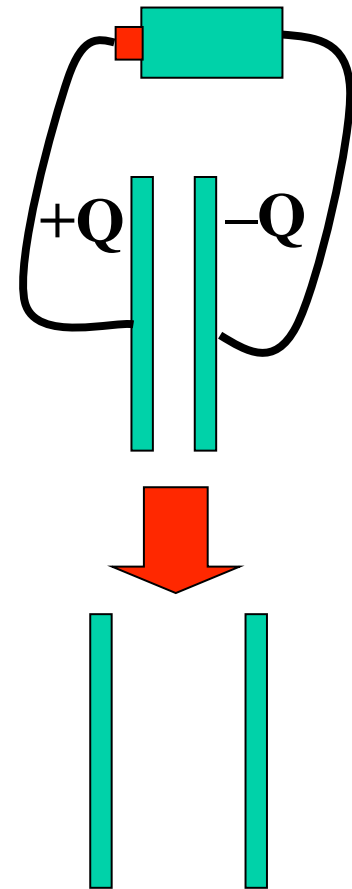
(Very Small!!)

Lesson: difficult to get large values of capacitance without special tricks!

Isolated Parallel Plate Capacitor

- A parallel plate capacitor of capacitance C is **charged** using a battery
- Charge = Q , potential difference = V
- Battery is then **disconnected**
- If the plate separation is **increased**, does **potential difference** V :

- (a) Increase
- (b) Remain the same
- (c) Decrease

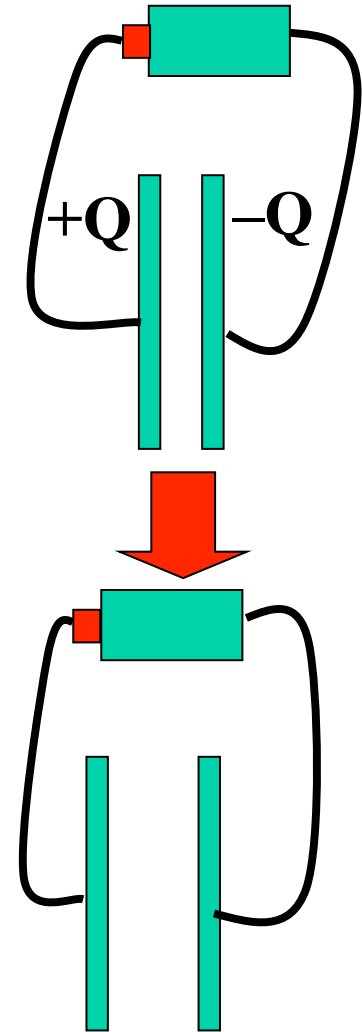


Charging a Parallel Plate Capacitor

- A parallel plate capacitor of capacitance C is **charged** using a battery
- Charge = Q , potential difference = V
- Plate separation is **increased** while battery remains connected

Does the **electric field** Inside:

- (a) Increase?
- (b) Remain the Same?
- (c) Decrease?



Spherical Capacitor

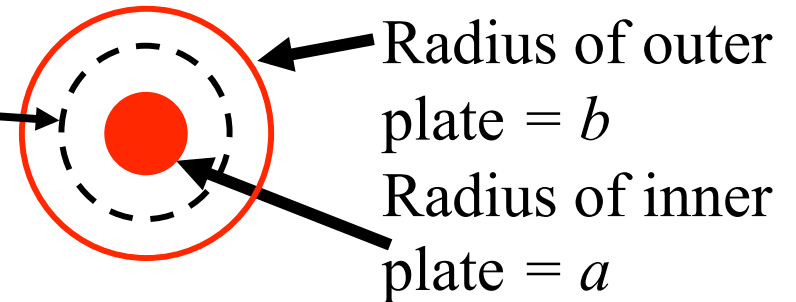
What is the electric field inside the capacitor? (Gauss' Law)

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Relate E to potential difference between the plates:

$$V = -\int_b^a \vec{E} \cdot d\vec{s} = \int_a^b \frac{kQ}{r^2} dr = \left[-\frac{kQ}{r} \right]_a^b$$

$$= kQ \left[\frac{1}{a} - \frac{1}{b} \right]$$



Concentric spherical shells:
Charge $+Q$ on inner shell,
 $-Q$ on outer shell

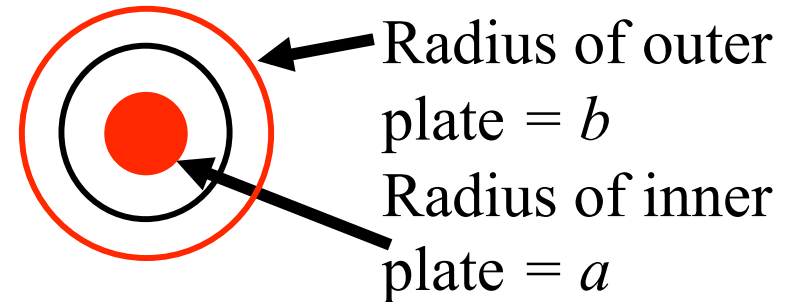
Spherical Capacitor

What is the capacitance?

$$C = Q/V$$

$$= \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\epsilon_0 \left[\frac{1}{a} - \frac{1}{b} \right]}}$$

$$= \frac{4\pi\epsilon_0 ab}{(b - a)}$$



Concentric spherical shells:
Charge $+Q$ on inner shell,
 $-Q$ on outer shell

Isolated sphere: let $b \gg a$,

$$C = 4\pi\epsilon_0 a$$

Summary

- Any two charged conductors form a **capacitor**
- **Capacitance:** $C = Q/V$
- Simple Capacitors:

Parallel plates: $C = \epsilon_0 A/d$

Spherical: $C = 4\pi \epsilon_0 ab/(b-a)$

