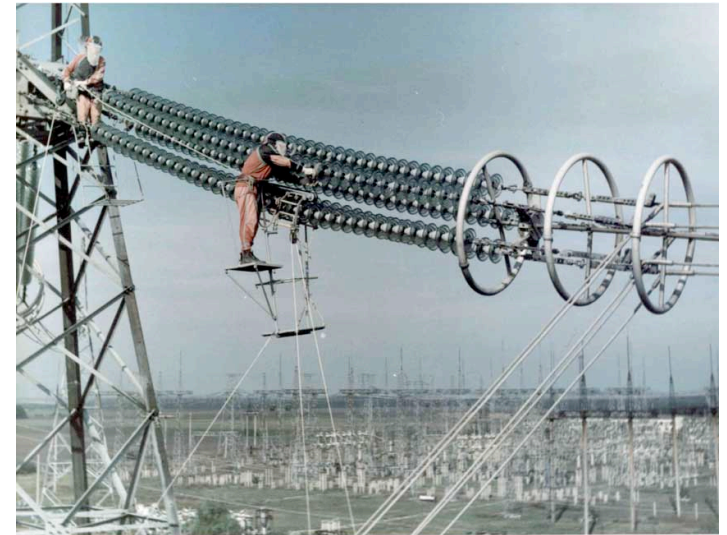


Physics 2102

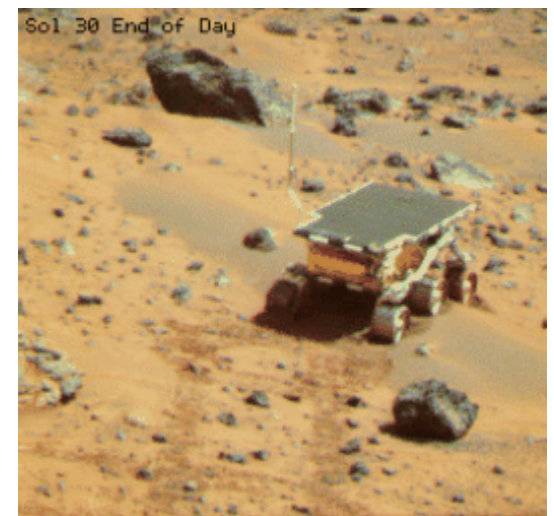
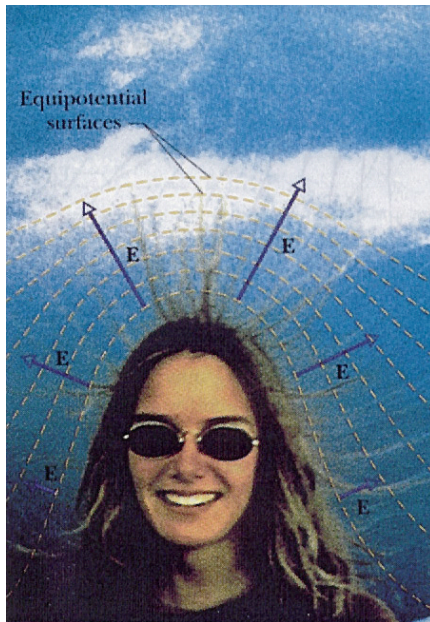
Christian Buth



# Physics 2102 Lecture 10

## Electric Potential 3

Version: 02/04/2009



# PHYS2102 FIRST MIDTERM EXAM!

6–7PM THU 05 FEB 2009

Buth's Sec. 6 in Lockett, Room 10

**YOU MUST BRING YOUR STUDENT ID!**

The exam will cover chapters 21 through 24, as covered in homework sets 1, 2, and 3. The formula sheet for the exam can be found here:

<http://www.phys.lsu.edu/classes/spring2009/phys2102/formulasheet.pdf>

**THERE WILL BE A REVIEW SESSION 6–7PM  
WED 04 FEB 2009 in Williams 103**

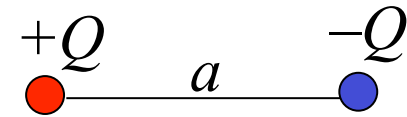
# Review

- Potential by a **point charge**:  $V=kq/r$
- Potential of a continuous **charge distribution**:  
 $V=\int k dq/r$
- Potential of a **dipole** (with angular dependence):  
$$V = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}$$
- Electric field follows from potential by **derivation**:

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

# Electric Potential Energy of a Dipole

What is the potential energy of a dipole?



- First bring charge  $+Q$ : **no work** involved, no potential energy.
- The charge  $+Q$  has created an electric potential **everywhere**,  
 $V(r) = kQ/r$
- The **work needed** to bring the charge  $-Q$  to a distance  $a$  from the charge  $+Q$  is  $W_{app} = U = (-Q)V = (-Q)(+kQ/a) = -kQ^2/a$
- **Dipole potential energy**:  $-kQ^2/a$
- We did **negative work** to build the dipole (and the electric field did positive work)

# Potential Energy of A System of Point Charges

- The electric potential energy of a **pair of point charges**

$$U_{ij} = q_i V_j = \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{r_{ij}}$$

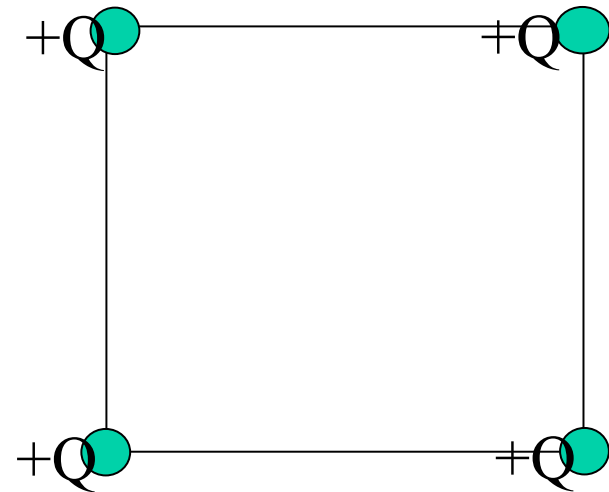
- Let  $U$  be the work required to **assemble** a system of  $n$  point charges one by one from infinity to its final position

$$U = \frac{1}{4\pi\epsilon_0} \sum_{\substack{i,j=1 \\ i < j}}^n \frac{q_i q_j}{r_{ij}}$$

- Each pair of charges is counted only **once**

# Potential Energy of A System of Charges

- 4 point charges (each  $+Q$  and equal mass) are connected by strings, forming a square of side  $L$
- If all four strings suddenly snap, what is the kinetic energy of each charge when they are very far apart?
- Use conservation of energy:
  - Final kinetic energy of all four charges = initial potential energy stored = energy required to assemble the system of charges



Do this from scratch!

# Potential Energy of A System of Charges: Solution

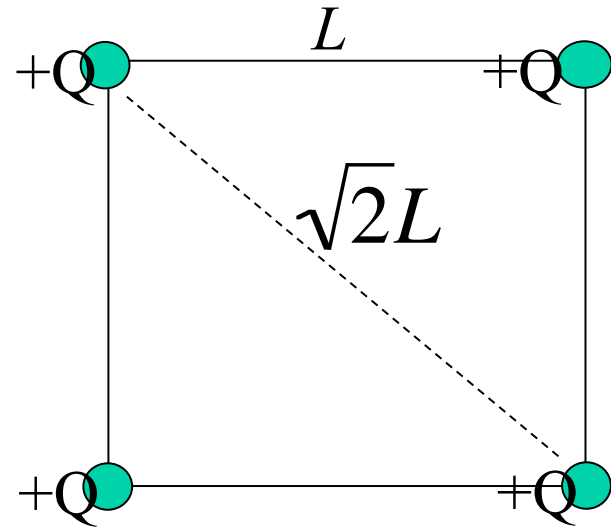
- No energy needed to bring in first charge:  $U_1=0$
- Energy needed to bring in 2nd charge:  $U_2 = QV_1 = \frac{kQ^2}{L}$

- Energy needed to bring in 3rd charge =

$$U_3 = QV = Q(V_1 + V_2) = \frac{kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}$$

- Energy needed to bring in 4th charge =

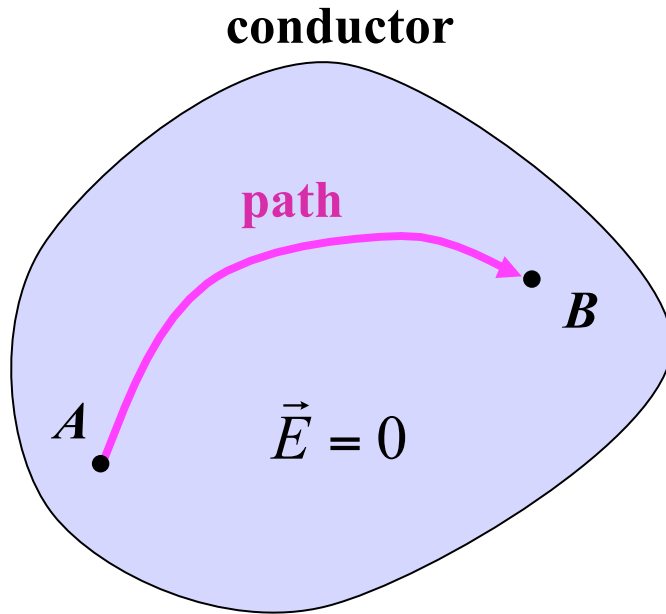
$$U_4 = QV = Q(V_1 + V_2 + V_3) = \frac{2kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}$$



Total potential energy is sum of all the individual terms shown on left hand side =  $\frac{kQ^2}{L} (4 + \sqrt{2})$

So, final kinetic energy of each charge =  $\frac{kQ^2}{4L} (4 + \sqrt{2})$

# Potential of an Isolated Conductor



- **Potential difference** between points A and B is

$$\Delta V = V_A - V_B = - \int_B^A \vec{E} \cdot d\vec{s}$$

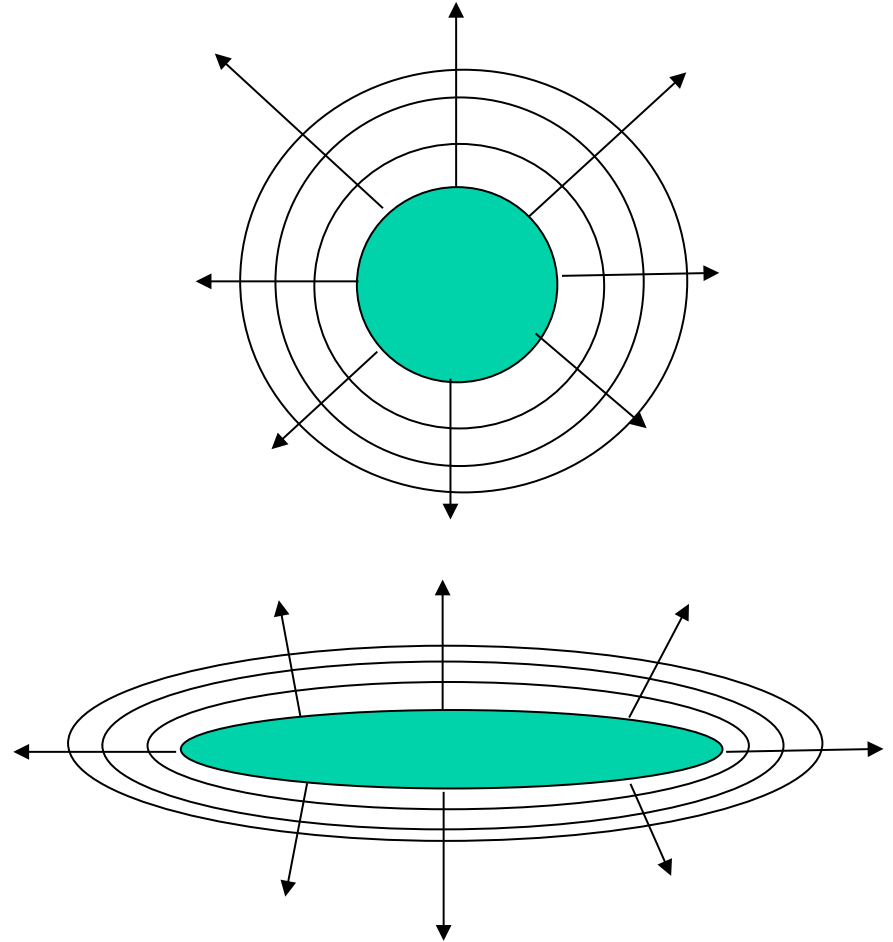
- Field in conductor is **zero**
- Thus  $V_A - V_B = 0$

A conductor is an equipotential surface.



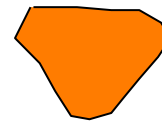
# Equipotentials and Conductors

- Conducting surfaces are **equipotentials**
- At surface of conductor,  $E$  is **normal** to surface
- **No work** needed to move a charge on a conductor surface



# Conductors change the field around them!

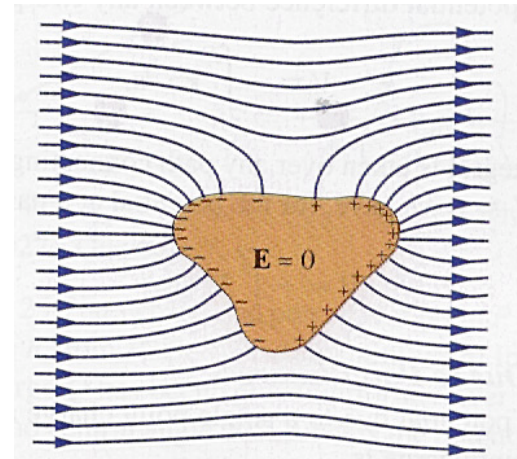
An uncharged conductor:



A uniform electric field:

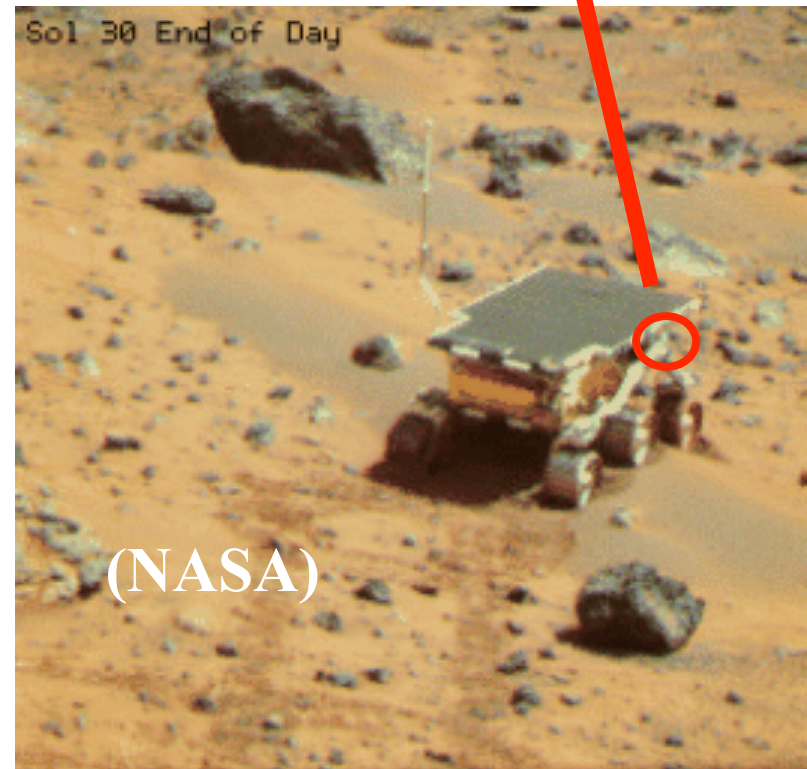
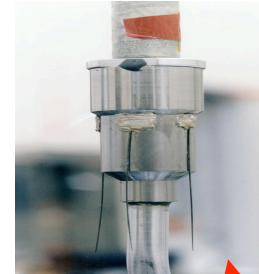


An uncharged conductor in the initially uniform electric field:



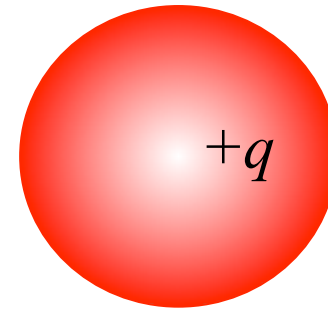
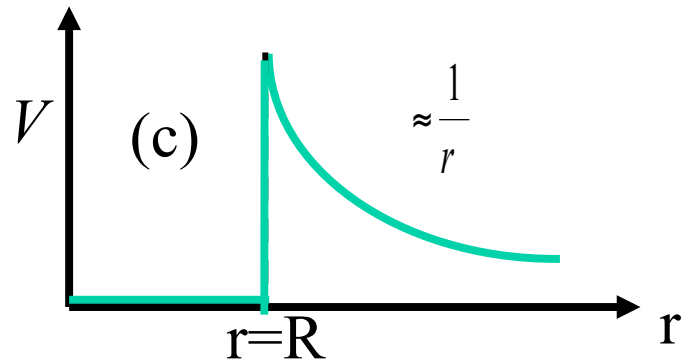
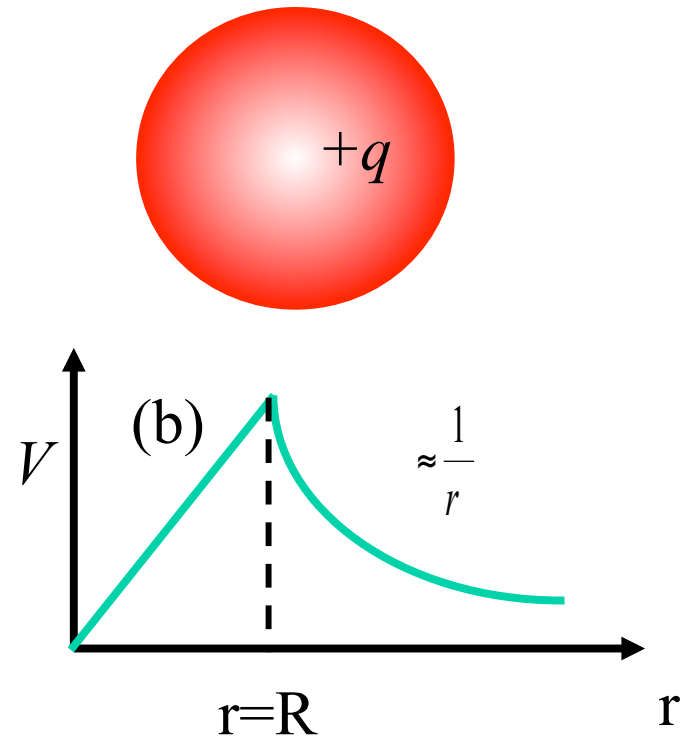
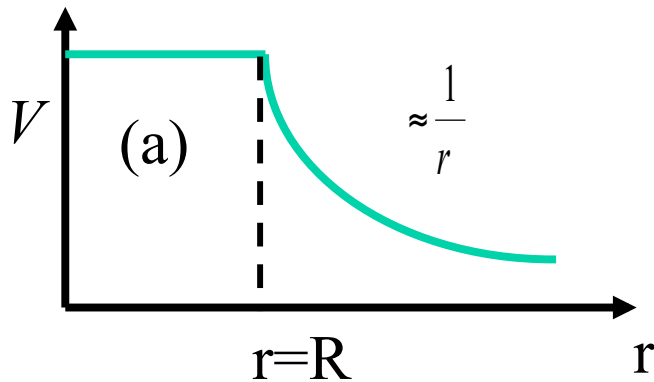
# “Sharp” conductors

- Charge density is **higher** at conductor surfaces that have small radius of curvature
- $E = \sigma/\epsilon_0$  for a conductor, hence **stronger** electric fields at sharply curved surfaces!
- Used for attracting or getting rid of charge:
  - Lightning rods
  - Van de Graaf -- metal brush transfers charge from rubber belt
  - Mars pathfinder mission -- tungsten points used to get rid of accumulated charge on rover (electric breakdown on Mars occurs at  $\sim 100$  V/m)

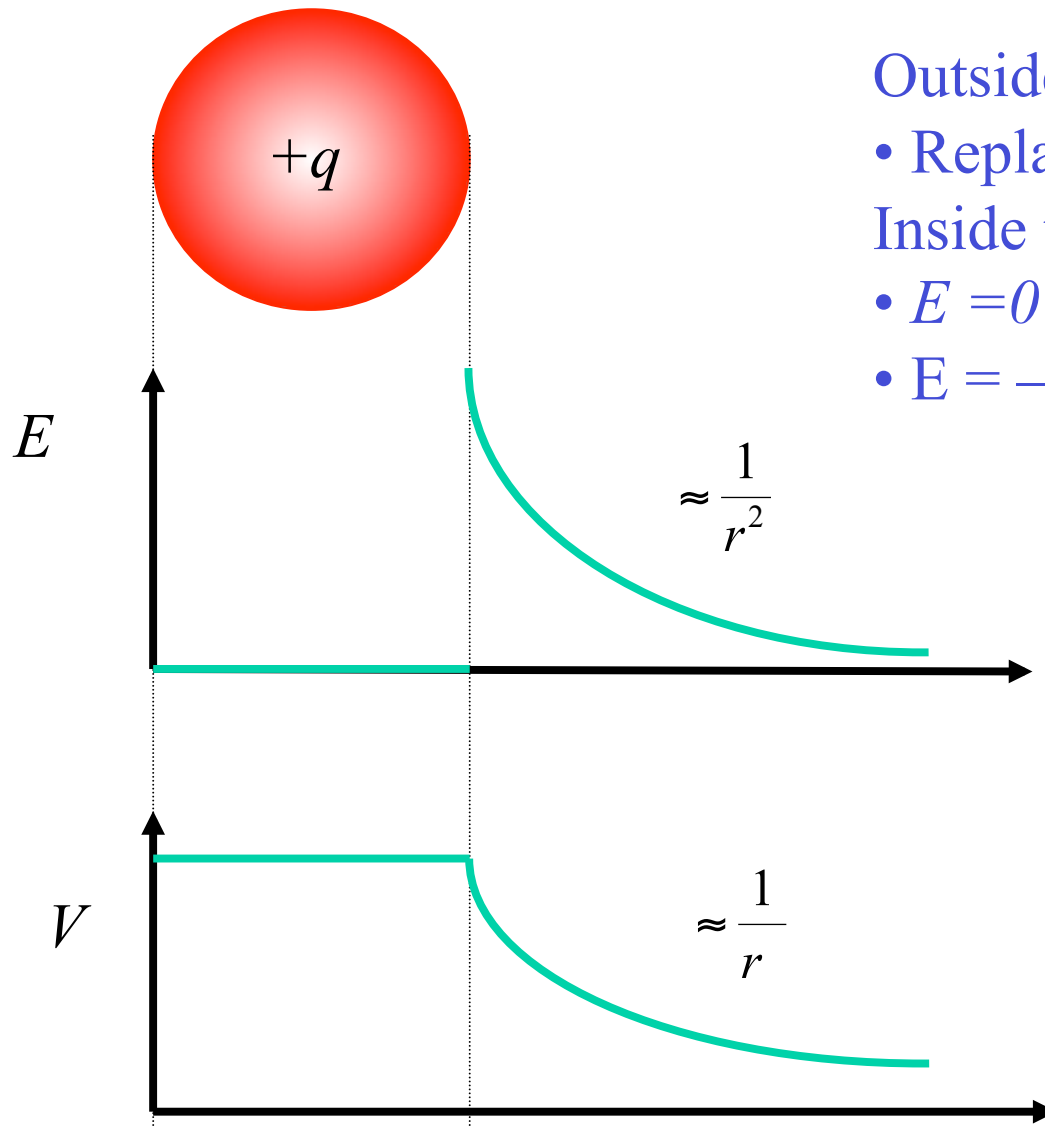


# Electric Field & Potential: Example

- Hollow **metal** sphere of radius  $R$  has a charge  $+q$
- Which of the following is the electric potential  $V$  as a function of distance  $r$  from center of sphere?



# Electric Field and Potential: Example



Outside the sphere:

- Replace by point charge!

Inside the sphere:

- $E = 0$  (Gauss' Law)
- $E = -dV/dr = 0 \Rightarrow V = \text{constant}$

$$\begin{aligned} E &= -\frac{dV}{dr} \\ &= -\frac{d}{dr} \left[ k \frac{Q}{r} \right] \\ &= k \frac{Q}{r^2} \end{aligned}$$

# Summary

- **Electric potential energy**: work used to build the system, charge by charge:

$$U = \frac{1}{4\pi\epsilon_0} \sum_{\substack{i,j=1 \\ i < j}}^n \frac{q_i q_j}{r_{ij}}$$

- Charges move to make surface of **conductors** equipotentials
- Thus conductors **change the field** around them!
- Charge density and electric field are higher on **sharp points** of conductor